6453

NRL Report 6365

Values for Scattering by Dielectric Spheres in the Region of the First Resonance

W. C. KELLER

Electromagnetic Materials Branch
Electronics Division

December 30, 1965

U.S. NAVAL RESEARCH LABORATORY
Washington, D.C.

ABSTRACT

The scattering intensity as a function of sphere size in wavelengths was plotted for H-plane scattering at 90° in the region of the first resonance. Graphs of κ (the dielectric constant) from 5 to 24 and tan δ (the loss tangent) from 0.001 to 0.2 are presented for nonmagnetic spheres in vacuo. The purpose of these data is to serve as a comparison standard in evaluating κ and tan δ by scattering measurements on a spherical sample. The experimental method is described in "A Method of Determining the Dielectric Constant Applicable to High Temperatures," published in the "Proceedings of the OSU-RTD Electromagnetic Window Symposium," Vol. III, June 1964.

PROBLEM STATUS

This is a final report on one phase of the problem; work is continuing on the problem.

AUTHORIZATION

NRL Problem P02-04
Projects SF 010-02-01-9296 and SF 010-02-01-9300

Manuscript submitted October 14, 1965.

Copies available from Clearinghouse for Federal Scientific and Technical Information (CFSTI) Sills Building, 5285 Port Royal Road, Springfield, Virginia 22151
$2.00

SCATTERING BY DIELECTRIC SPHERES IN THE REGION OF THE FIRST RESONANCE

INTRODUCTION

The general solution for a plane wave incident on a homogeneous isotropic sphere was given by Mie (1). The intensity of the radiation scattered by such a sphere is a function of its size, the index of refraction, the scattering angle, and the intensity, frequency, and polarization of the incident wave. Throughout this discussion, a sphere of unit permeability in vacuo is assumed.

This report is an intensive investigation covering a selected range of the above variables. It is limited to H-plane scattering at 90° in the region of the first resonance. The complex index of refraction is given in terms of the dielectric constant K and loss tangent δ for the following values: K = 5.0 (0.2) 12.8* and 13 (1) 24 and tan δ = 0.001, 0.01, and 0.1; K = 6.0 (0.2) 11.8 and 12 (1) 24 and tan δ = 0.04 and 0.07; and K = 5 (1) 16 and tan δ = 0.2. The data are presented in graphic form for convenience in evaluating experimental scattering measurements. This method of measuring K and tan δ is discussed elsewhere (3).

RADAR CROSS SECTION OF DIELECTRIC SPHERES

For the incident electric vector perpendicular to the plane of scattering (H-plane) (4)

$$I = \frac{I_0}{k^2 r^2} \, |S_1(\theta)|^2 \qquad (1)$$

where I_0 is the intensity of the incident wave, I is the intensity of the scattered wave, $k = 2\pi/\lambda$, r is the range, and θ is the angle between the direction of propagation of the incident and scattered wave. $S_1(\theta)$ is the complex scattering function

$$S_1(\theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \left[\frac{a_n P_n^1(\cos\theta)}{\sin\theta} + b_n \frac{d}{d\theta} P_n^1(\cos\theta) \right] \qquad (2)$$

in which P_n^1 is the associated Legendre polynomial and a_n and b_n, the Mie coefficients, are functions of ka and m, where a is the radius and m is the index of refraction, given by

$$m = \sqrt{K(1 - i \tan\delta)} \,.$$

For H-plane scattering at 90° the radar cross section of a sphere is given by

$$\sigma = 4\pi r^2 \frac{I}{I_0} = \frac{4\pi}{k^2} \left| S_1\left(\frac{\pi}{2}\right) \right|^2 . \qquad (3)$$

---

*This notation indicates that values of K range from 5.0 to 12.8 with an interval of 0.2 between values (2).

It is convenient to consider the ratio of σ for a dielectric sphere (σ_d) to σ for a conducting sphere (σ_c). For σ_d/σ_c in db

$$10 \log \frac{\sigma_d}{\sigma_c} = 10 \log \left|S_1\left(\frac{\pi}{2}\right)\right|_d^2 - 10 \log \left|S_1\left(\frac{\pi}{2}\right)\right|_c^2. \tag{4}$$

EFFECT OF VARIOUS PARAMETERS ON SCATTERING INTENSITY

In Figs. 1 through 59, 10 log $|S_1(\pi/2)|^2$ is plotted as a function of ka. The curves in Figs. 1 through 58 are plotted using as parameters the values of K and tan δ that were stated in the introduction. Figure 59 is the case for tan $\delta = \infty$. The lowest value of the loss tangent presented is 10^{-3} . Below this value the effect of loss on $S_1(\pi/2)$ is insignificant. Over the range of K and ka under discussion, curves for tan $\delta = 0$ would almost coincide with tan $\delta = 10^{-3}$ except for a very narrow region on the first resonance hump where the scattering for lossless spheres may be as much as 1 db greater. The range of ka was chosen to include the first resonant dip.

Reference to the data illustrates that over a certain range of the dielectric constant and loss tangent a small change in these parameters results in a large change in $S_1(\pi/2)$. The larger this change, the more detailed information we can obtain from scattering measurements. For $K < 6$ and tan $\delta \leq 10^{-2}$ the resonance dip is shallow. Increasing the loss tangent has two effects: the resonance becomes deeper, and the rate of change in $S_1(\pi/2)$ as a function of tan δ is greater. For $K \gtrsim 8$ the dip in the curve is sharpest between tan $\delta = 10^{-1}$ and tan $\delta = 10^{-2}$. For higher values than tan $\delta = 10^{-1}$ the resonance disappears. If K is increased in constant increments, the distance between curves progressively becomes slightly less. If increments are chosen in percent of dielectric constant, this decrease in distance is small. The limitations on polarization, scattering angle, and size parameter are stated above. Investigation of scattering for other values of these parameters may extend the range of K and tan δ for which useful information is obtainable.

COMPUTATION METHOD AND ACCURACY

The Mie coefficients may be expressed in terms of spherical Bessel functions. If tan $\delta \neq 0$, the argument is complex. Since tables of Bessel functions for complex argument are not available, it was necessary to compute them. The Mie coefficients were programmed on the NAREC (Naval Research Electronic Computer). The accuracy of a_n and b_n was checked to four significant figures by comparing them with values reported in the literature. It is believed they are accurate to several more places.

For the small values of ka used, only the first few terms in the complex scattering function are significant. Four terms were used for the values of $S_1(\pi/2)$ reported. The distance between computed points varies widely along the curves. The smallest interval for ka was 0.01. Data points were placed close enough together to show all significant wiggles on the curves. Except for very sharp peaks and dips it is estimated that all errors are less than 1/2 db. In general, the data are accurate to within 0.1 db.

ACKNOWLEDGMENT

The author wishes to thank Mr. R. E. McGill, Applied Mathematics Staff, who programmed the Mie coefficients on the NAREC.

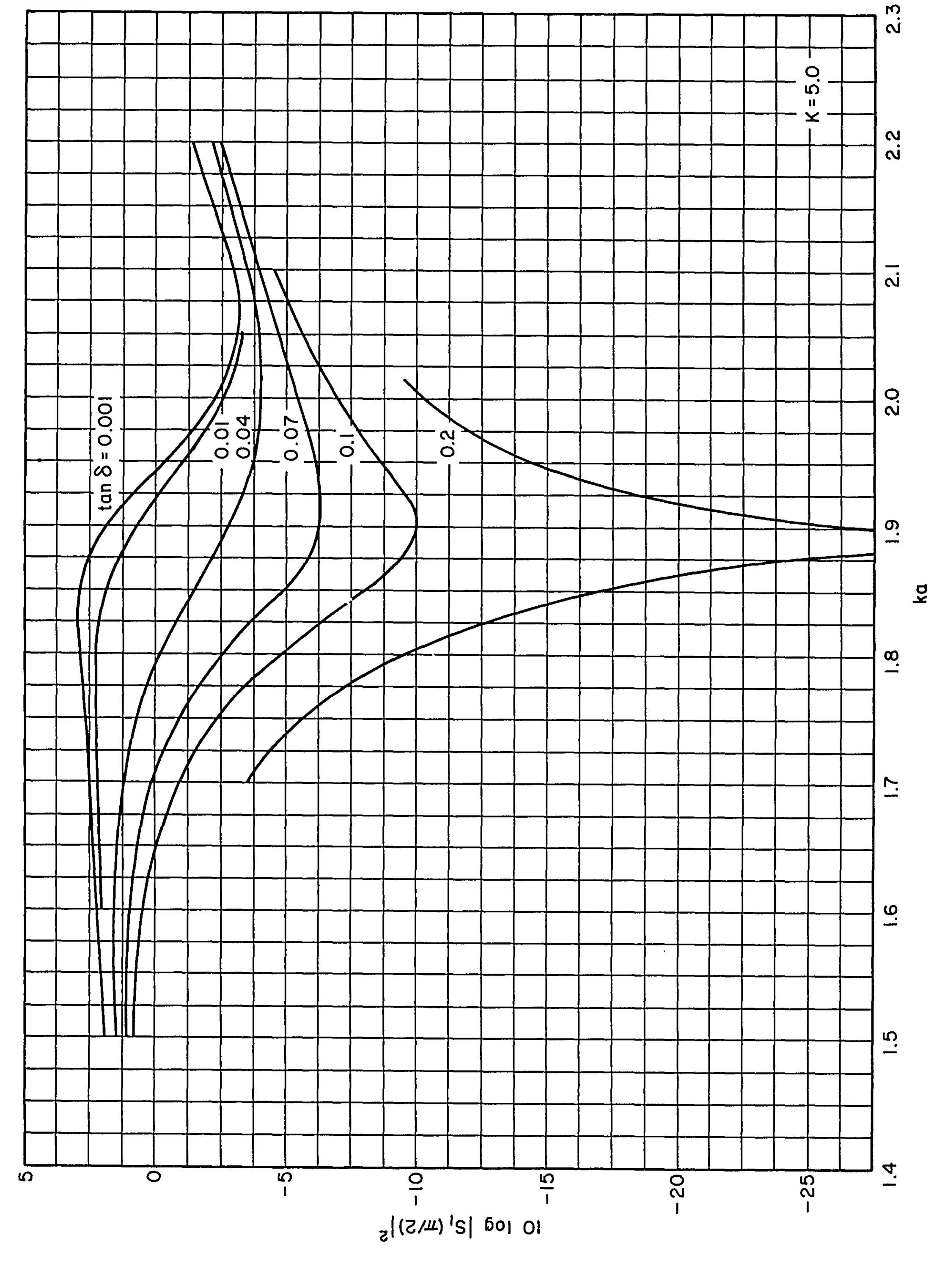
REFERENCES

1. Mie, G., Ann. Physik 25:377 (1908)

2. Van de Hulst, H.C., "Light Scattering by Small Particles," Ch. 10, p. 165, New York: Wiley, 1957

3. Keller, W.C., "A Method of Determining the Dielectric Constant Applicable to High Temperatures," Proc. of OSU-RTD Electromagnetic Window Symposium, Vol. III, Columbus, Ohio, June 1964

4. Van de Hulst, H.C., "Light Scattering by Small Particles," Ch. 4, p. 35 and Ch. 9, pp. 123-126, New York:Wiley, 1957



tan δ = 0.001
0.01
0.04
0.07
0.1
0.2
K = 5.0
ka
10 log |S₁(π/2)|²
5
0
-5
-10
-15
-20
-25
1.4
1.5
1.6
1.7
1.8
1.9
2.0
2.1
2.2
2.3

Figure 1

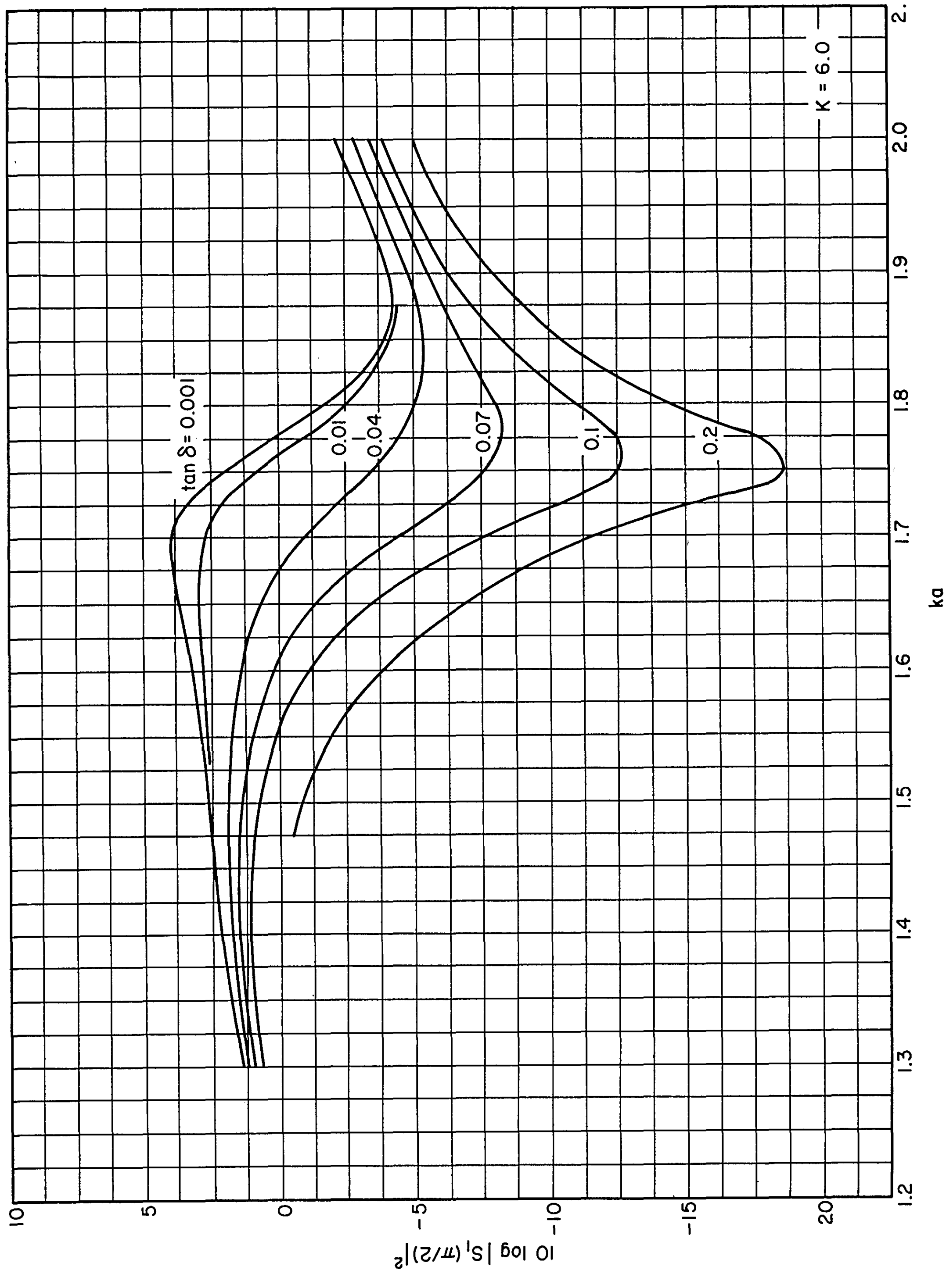
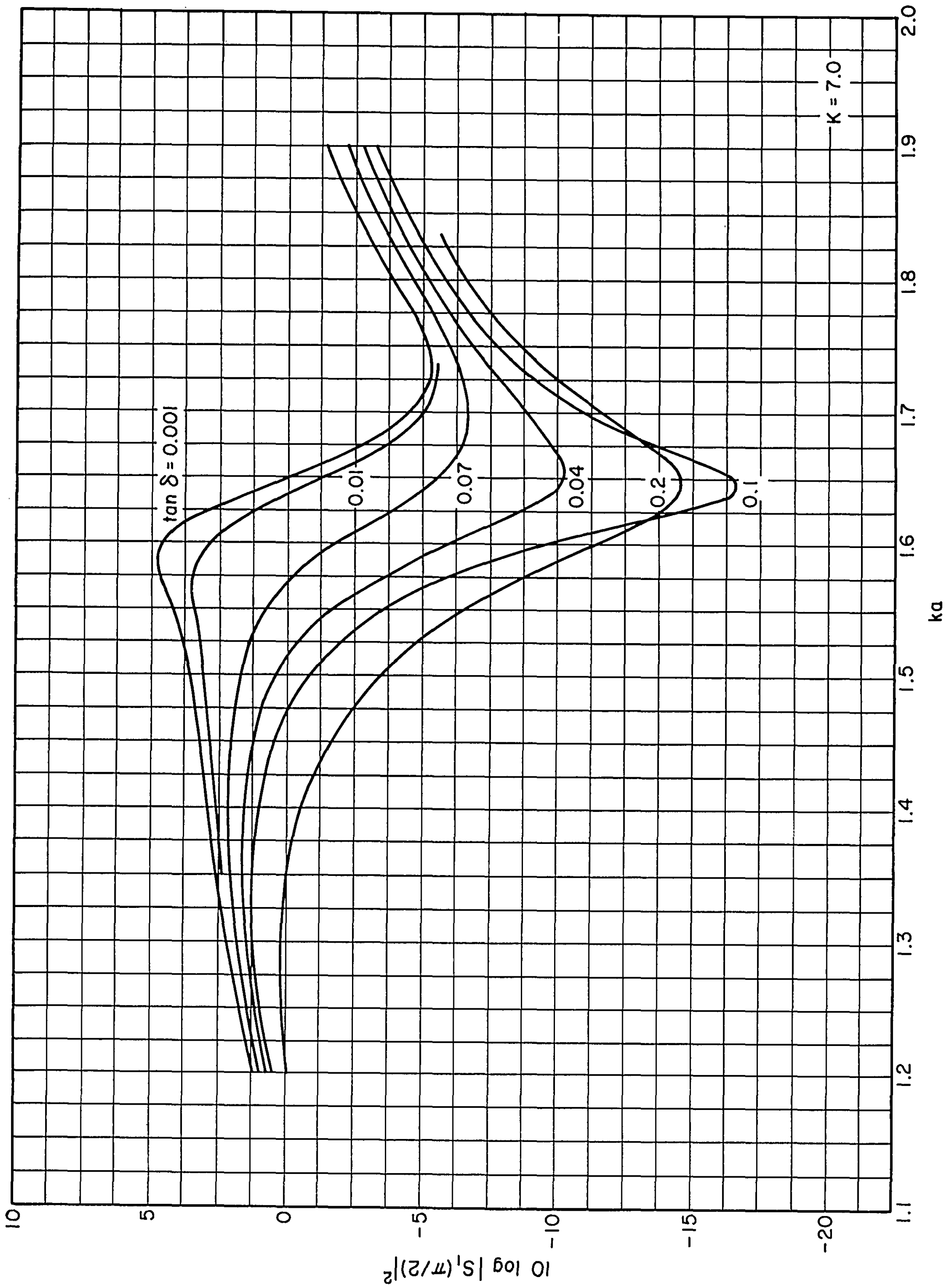

K = 6.0
tan δ = 0.001
0.01
0.04
0.07
0.1
0.2
ka
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
2.1
10 log |S₁(π/2)|²
10
5
0
−5
−10
−15
−20

Figure 2



tan δ = 0.001
0.01
0.07
0.04
0.2
0.1
K = 7.0
10
5
0
-5
-10
-15
-20
10 log |S₁(π/2)|²
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
ka

Figure 3

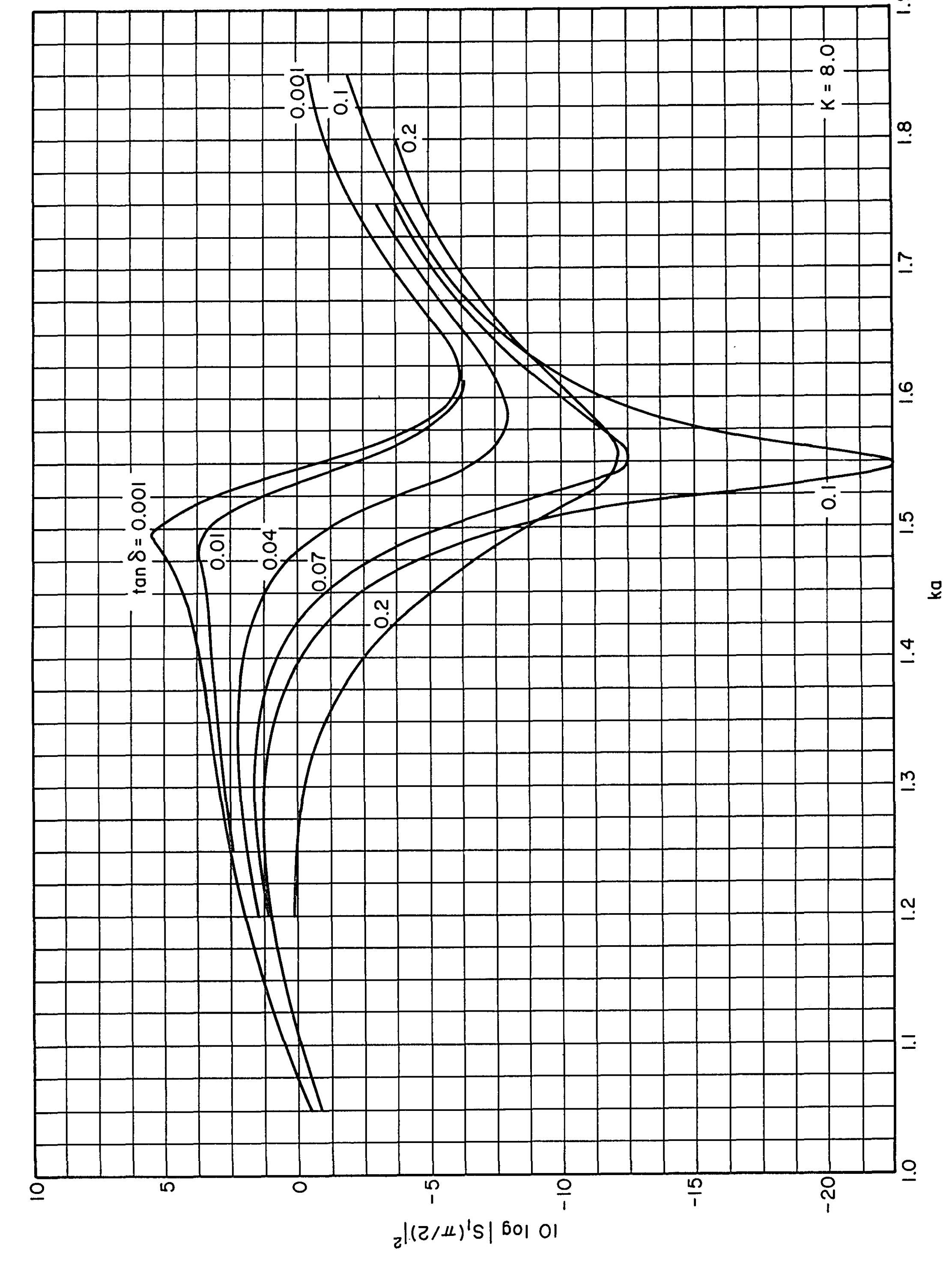

tan δ = 0.001
0.01
0.04
0.07
0.1
0.2
0.001
0.1
0.2
K = 8.0
ka
10 log |$S_1(\pi/2)$|^2
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
10
5
0
-5
-10
-15
-20

Figure 4

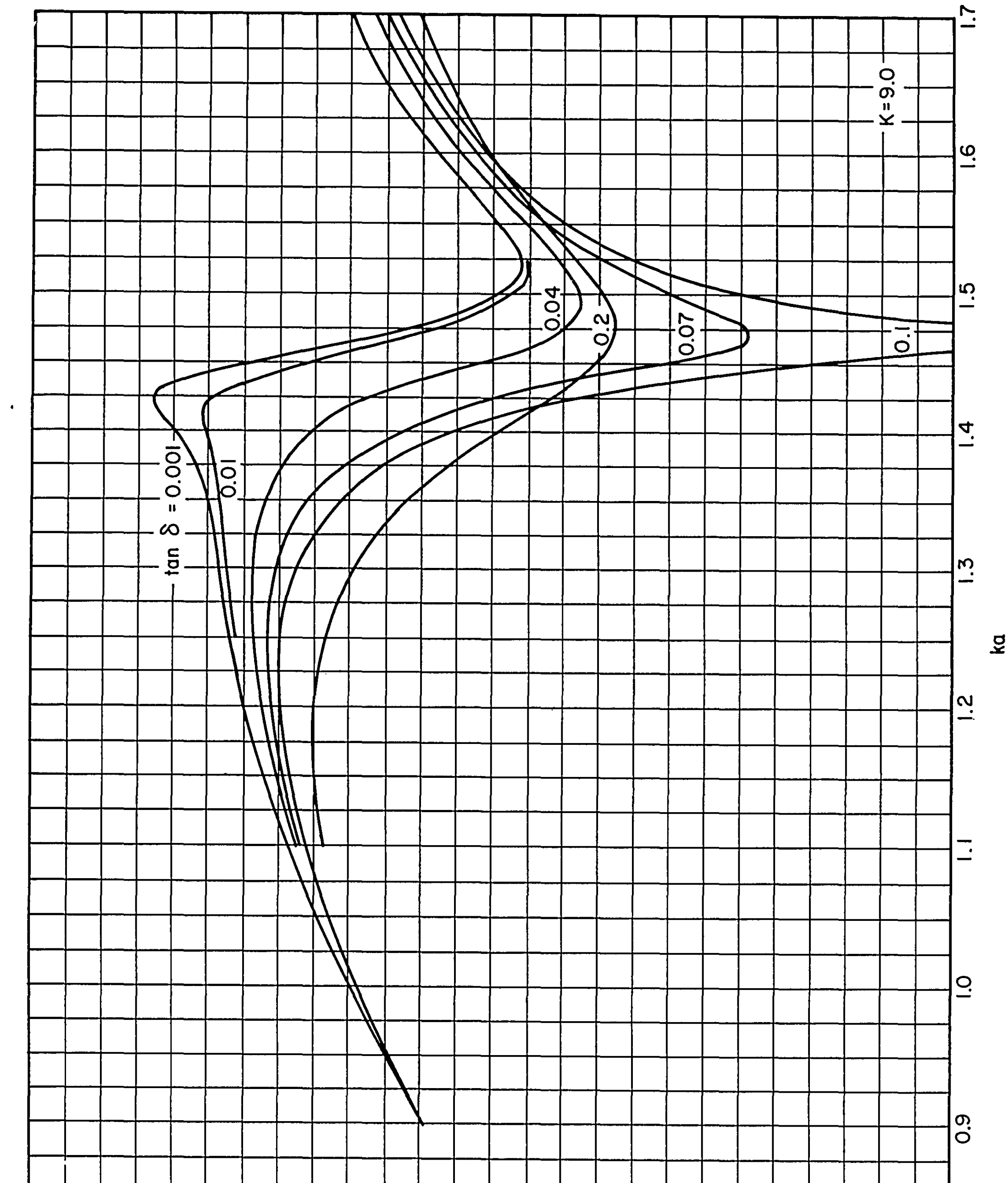

10 log $|S_1(\pi/2)|^2$
ka
tan δ = 0.001
0.01
0.04
0.2
0.07
0.1
K=9.0
10
5
0
−5
−10
−15
−20
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7

Figure 5

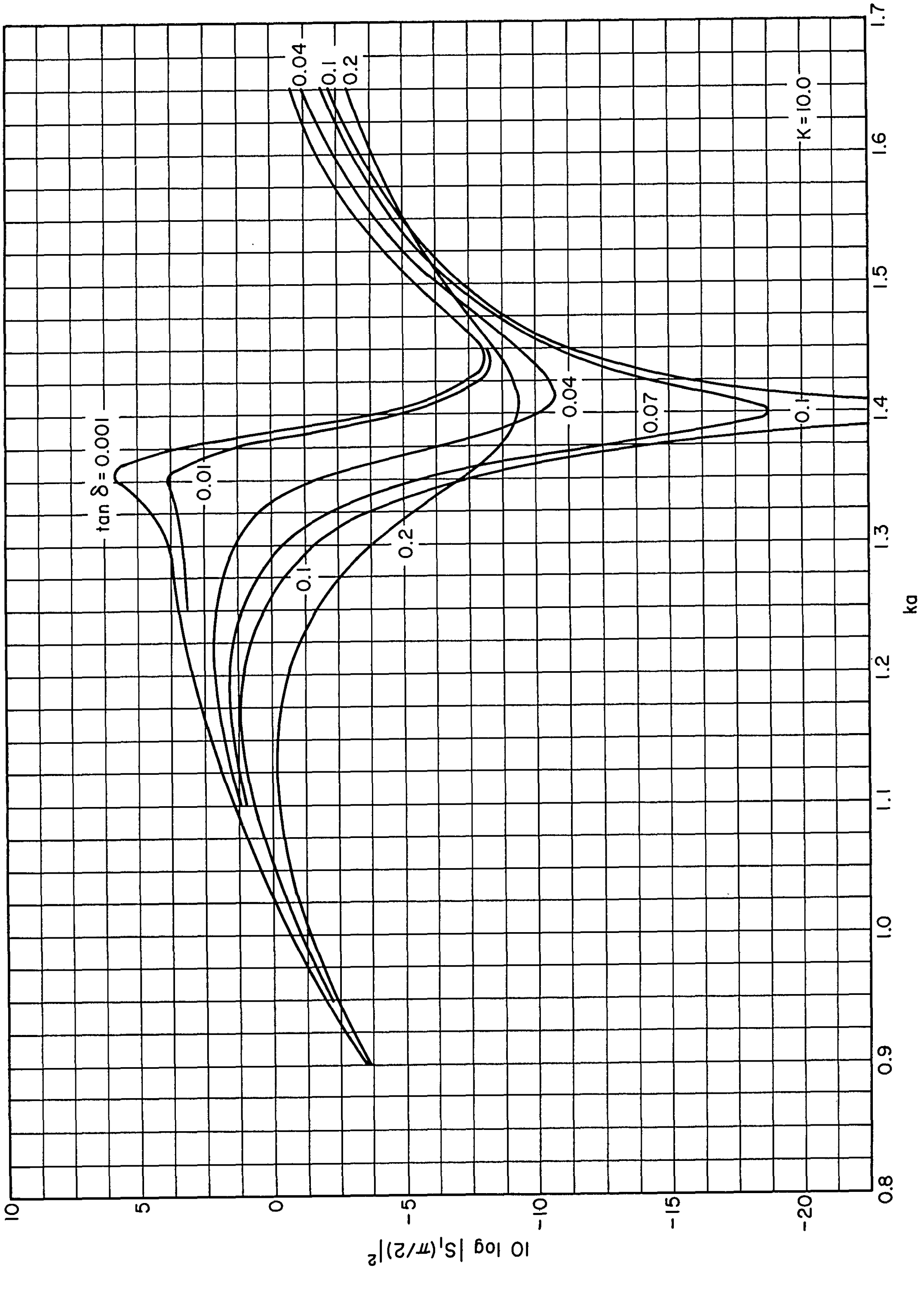
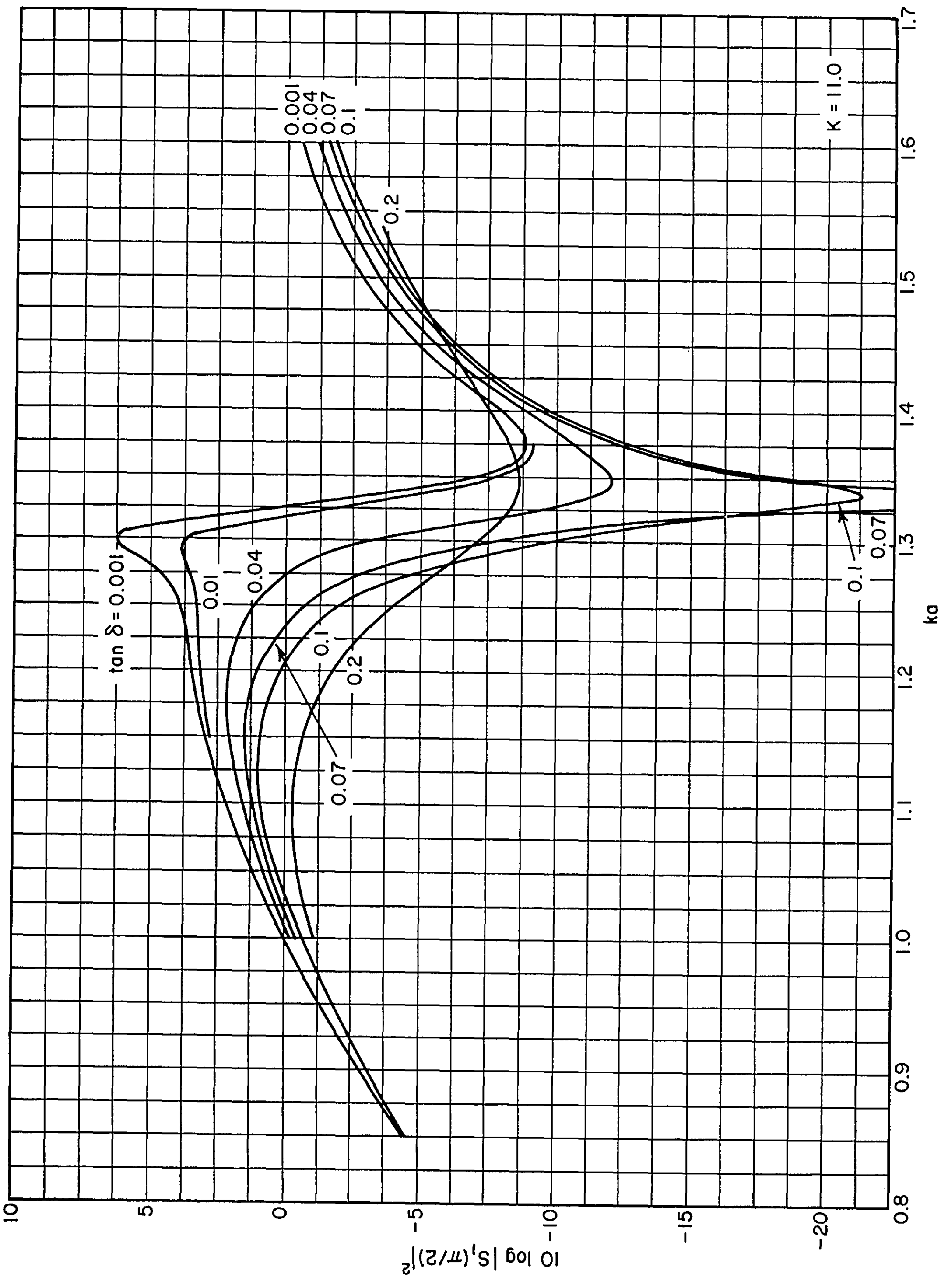

tan δ = 0.001
0.01
0.1
0.2
0.04
0.07
0.1
0.04
0.1
0.2
K = 10.0
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
ka
10
5
0
-5
-10
-15
-20
10 log |S1(π/2)|²

Figure 6



tan δ = 0.001
0.01
0.04
0.07
0.1
0.2
0.001
0.04
0.07
0.1
0.2
0.1
0.07
K = 11.0
ka
10 log |S1(π/2)|²
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
10
5
0
−5
−10
−15
−20

Figure 7

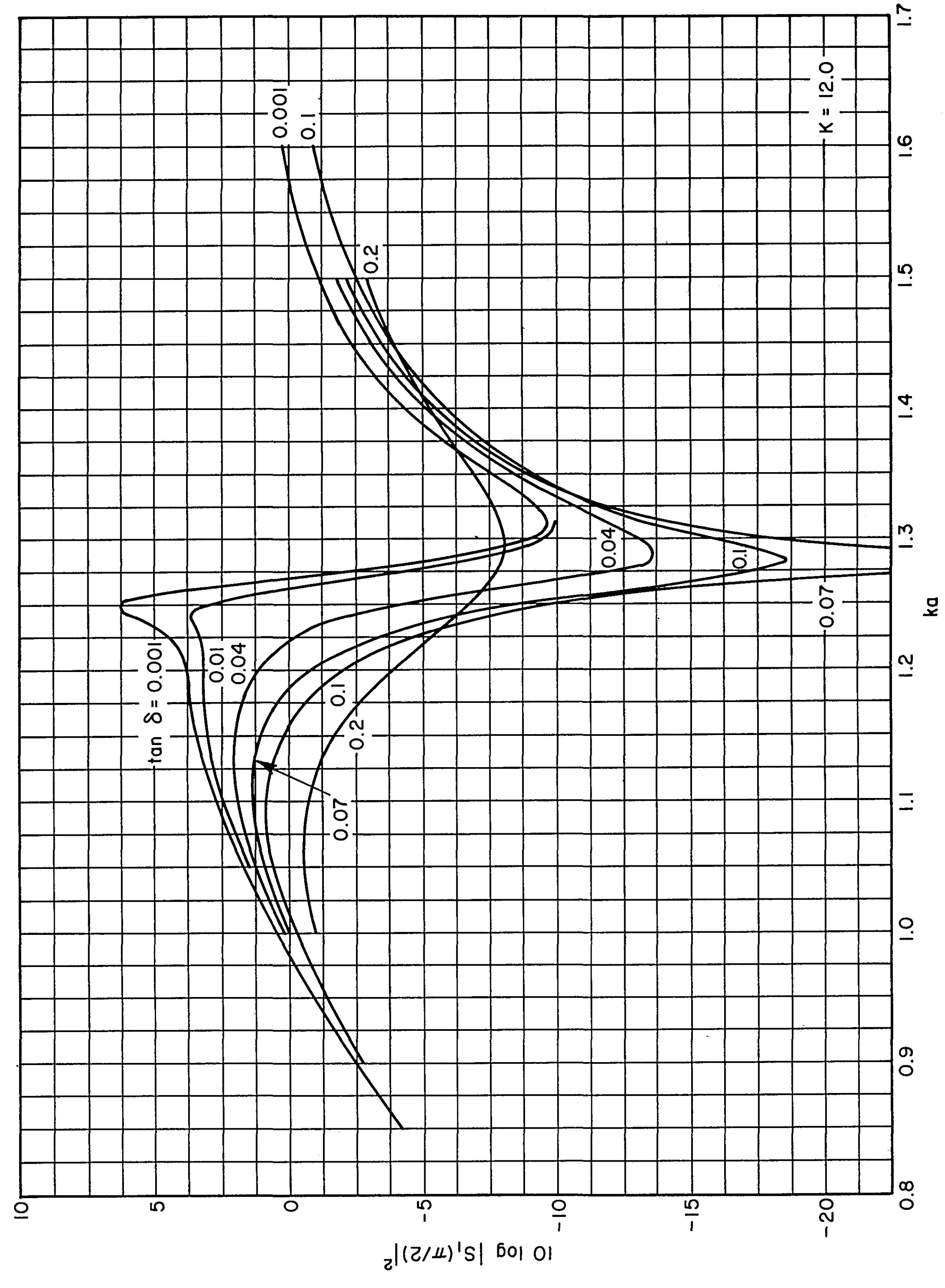

K = 12.0
tan δ = 0.001
0.01
0.04
0.07
0.1
0.2
0.001
ka
10 log |S₁(π/2)|²
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
10
5
0
-5
-10
-15
-20

Figure 8

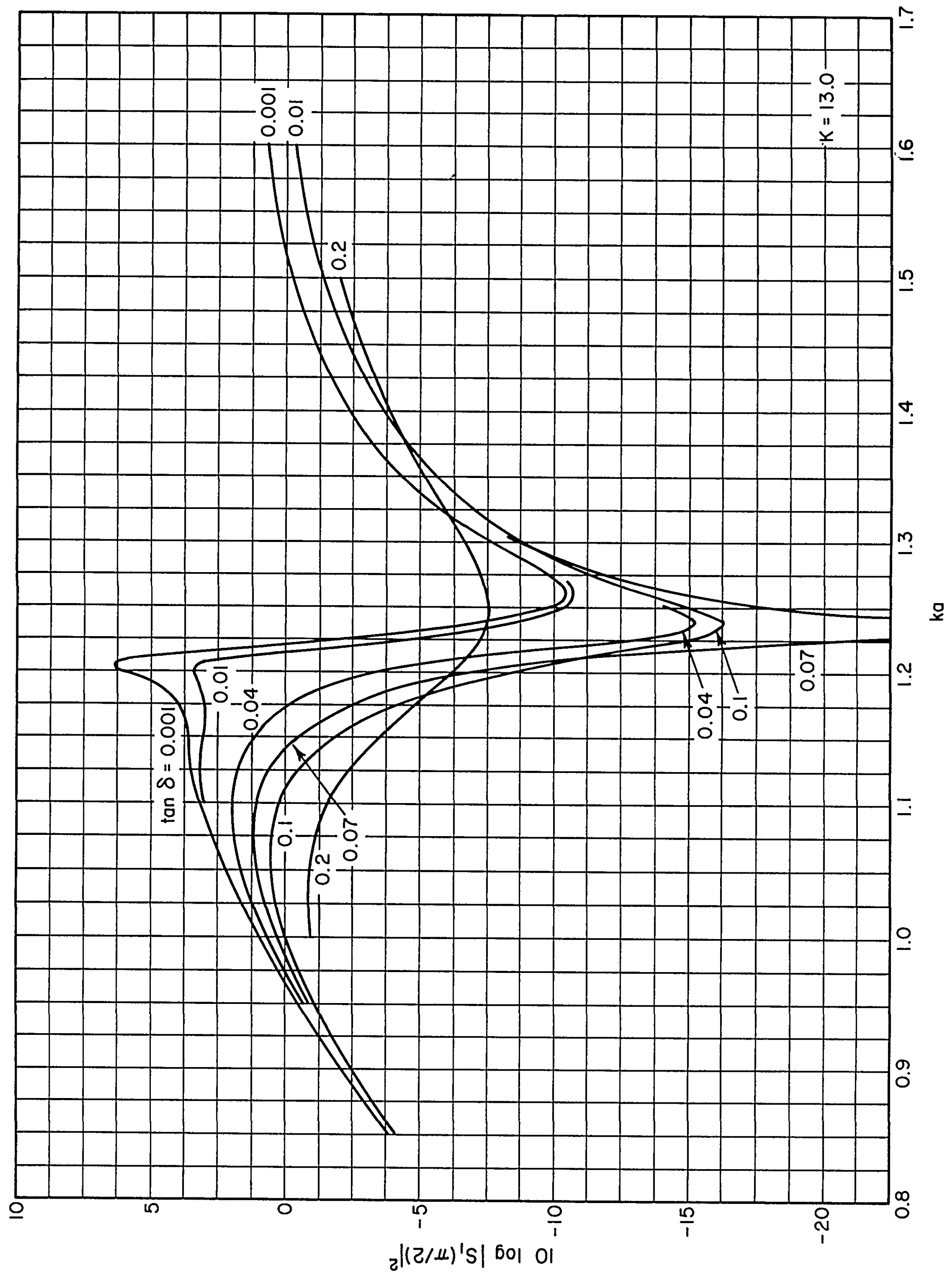
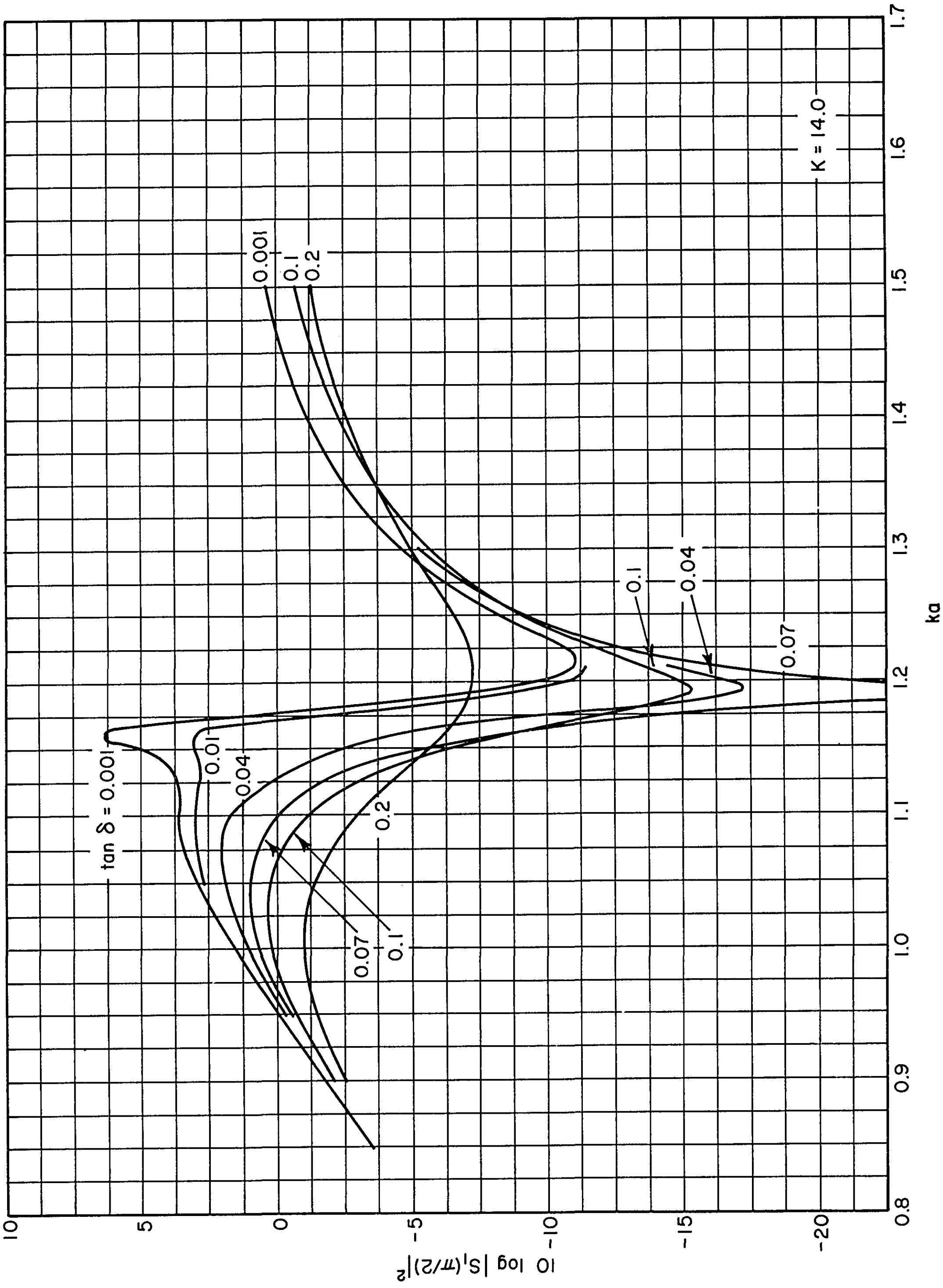


Figure 9



tan δ = 0.001
0.01
0.04
0.07
0.1
0.2
0.001
0.1
0.2
0.1
0.04
0.07
K = 14.0
ka
10 log |S₁(π/2)|²
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
10
5
0
−5
−10
−15
−20

Figure 10

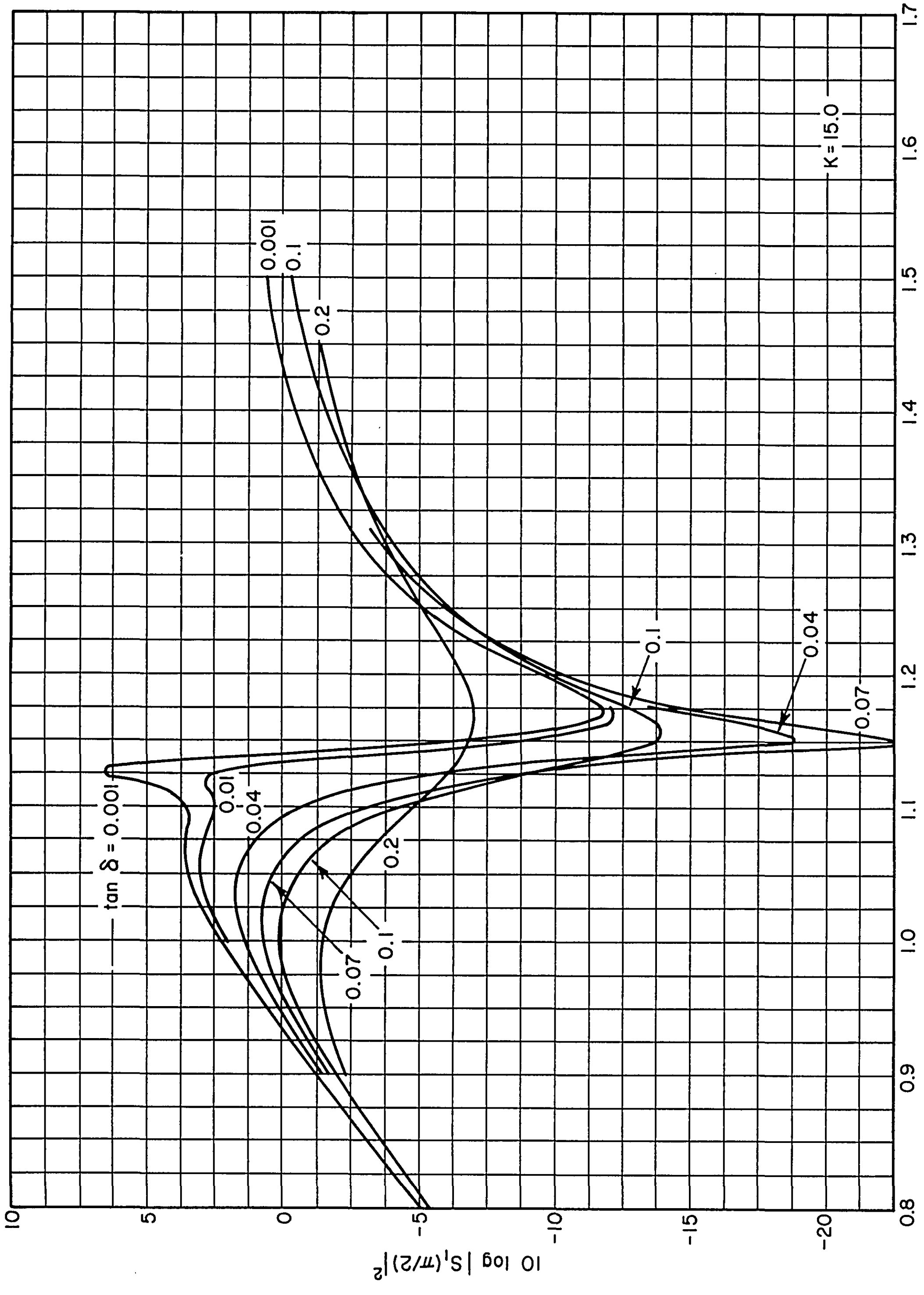

10 log $|S_1(\pi/2)|^2$
10
5
0
-5
-10
-15
-20
ka
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
K = 15.0
tan δ = 0.001
0.01
0.04
0.07
0.1
0.2
0.001
0.1
0.2
0.1
0.04
0.07

Figure 11

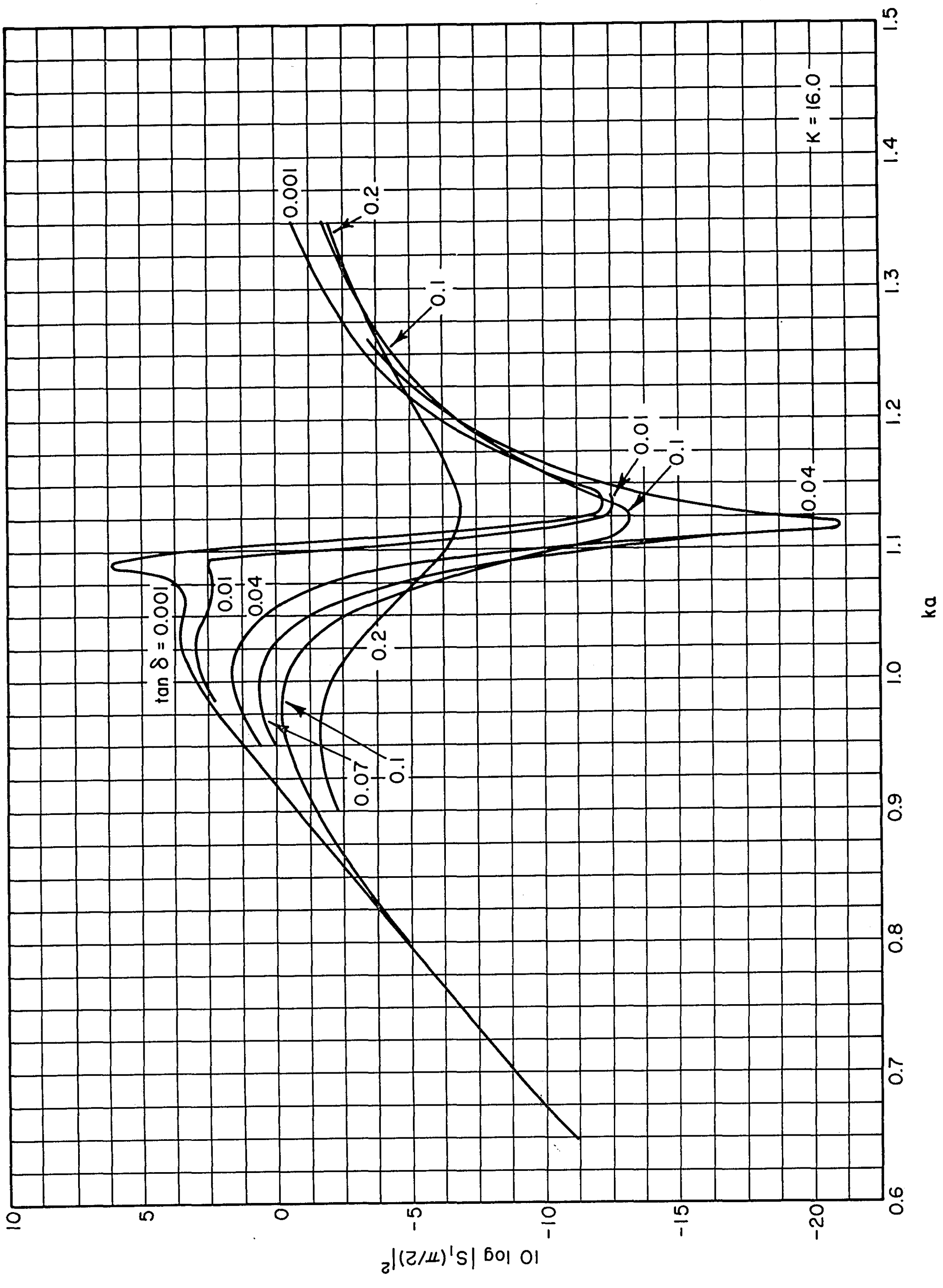

K = 16.0
tan δ = 0.001
0.01
0.04
0.07
0.1
0.2
0.001
ka
10 log |S1(π/2)|²
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
10
5
0
-5
-10
-15
-20

Figure 12

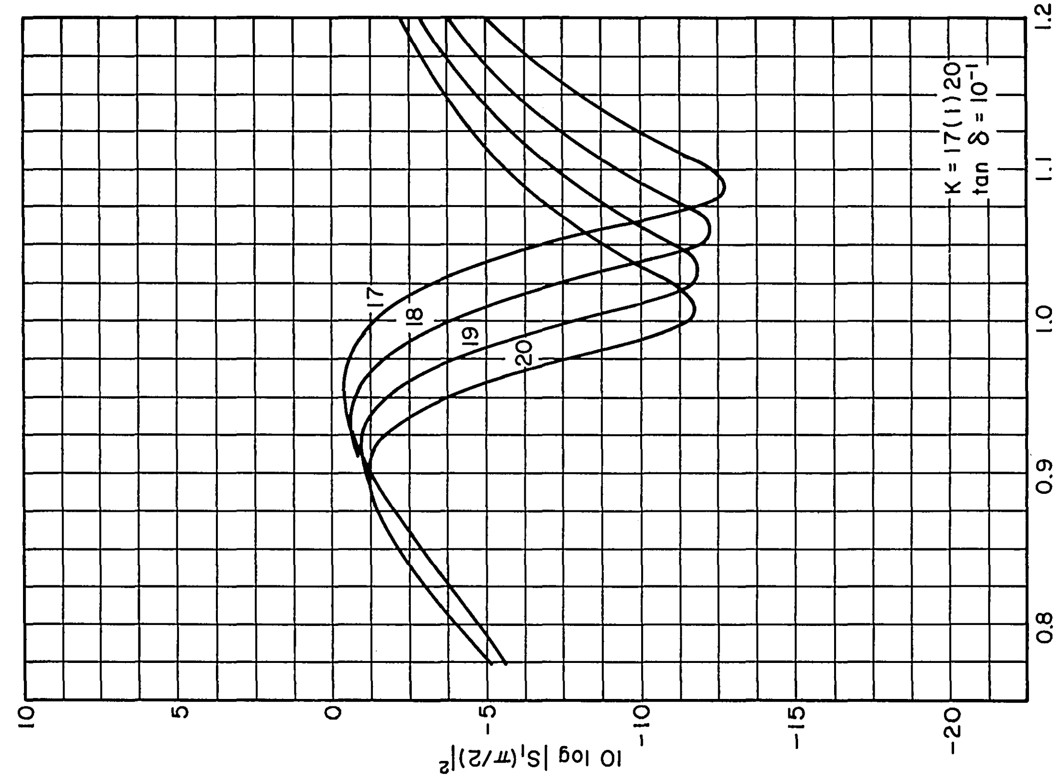


Figure 13

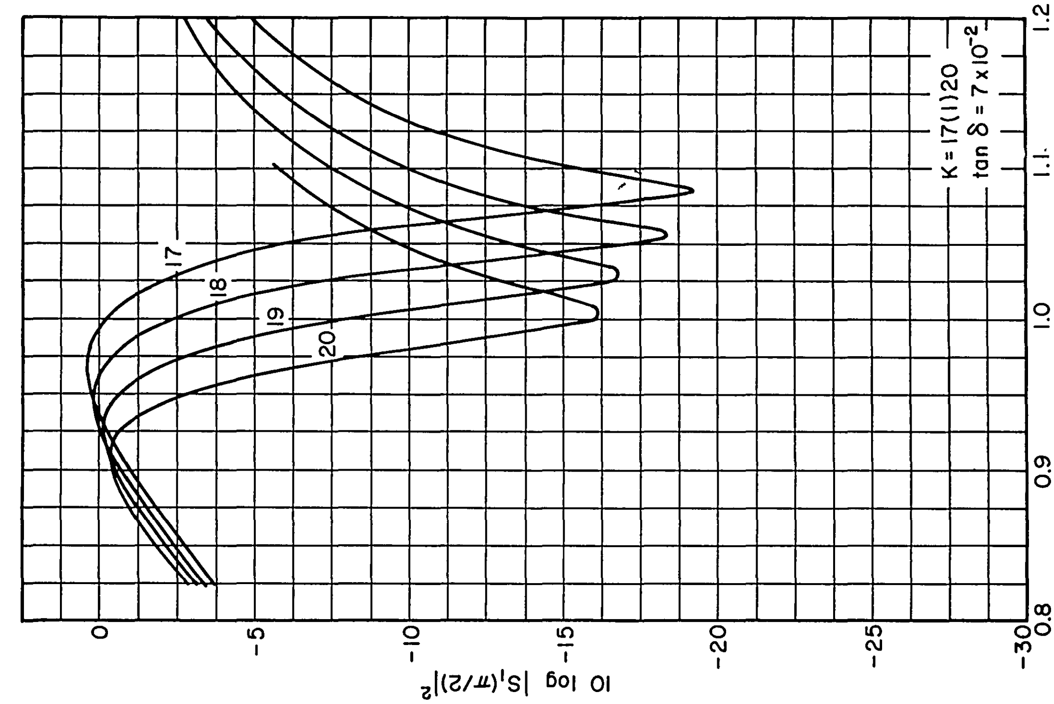


Figure 14

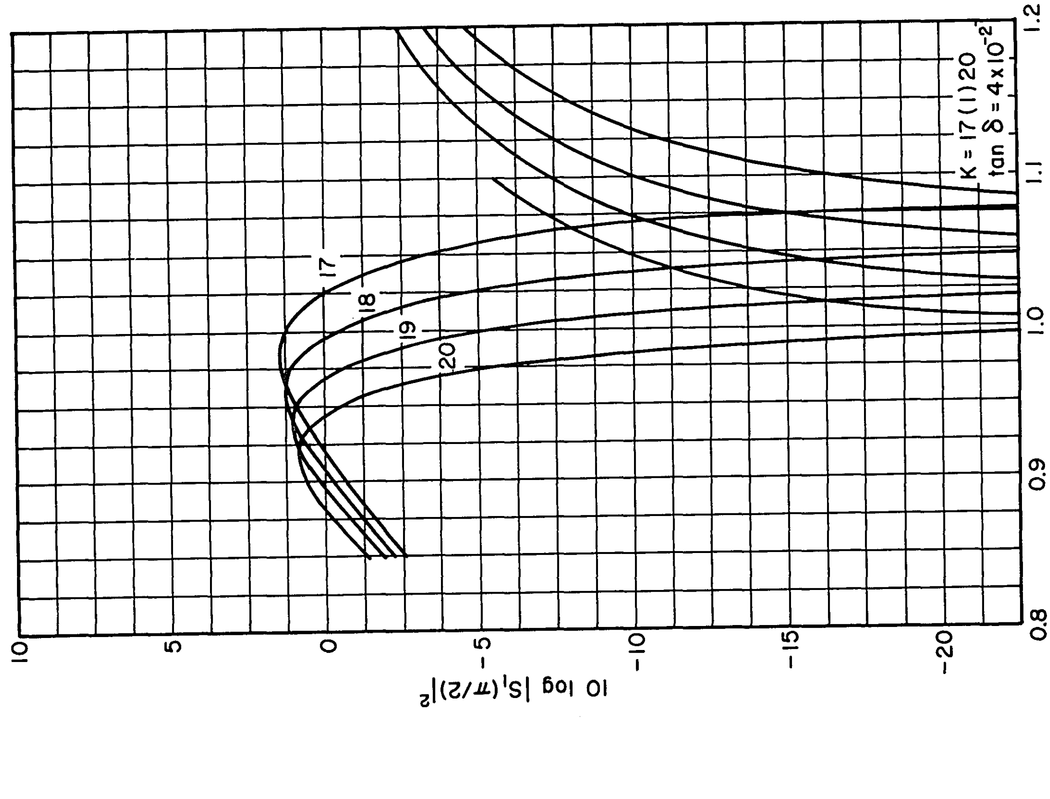


Figure 15

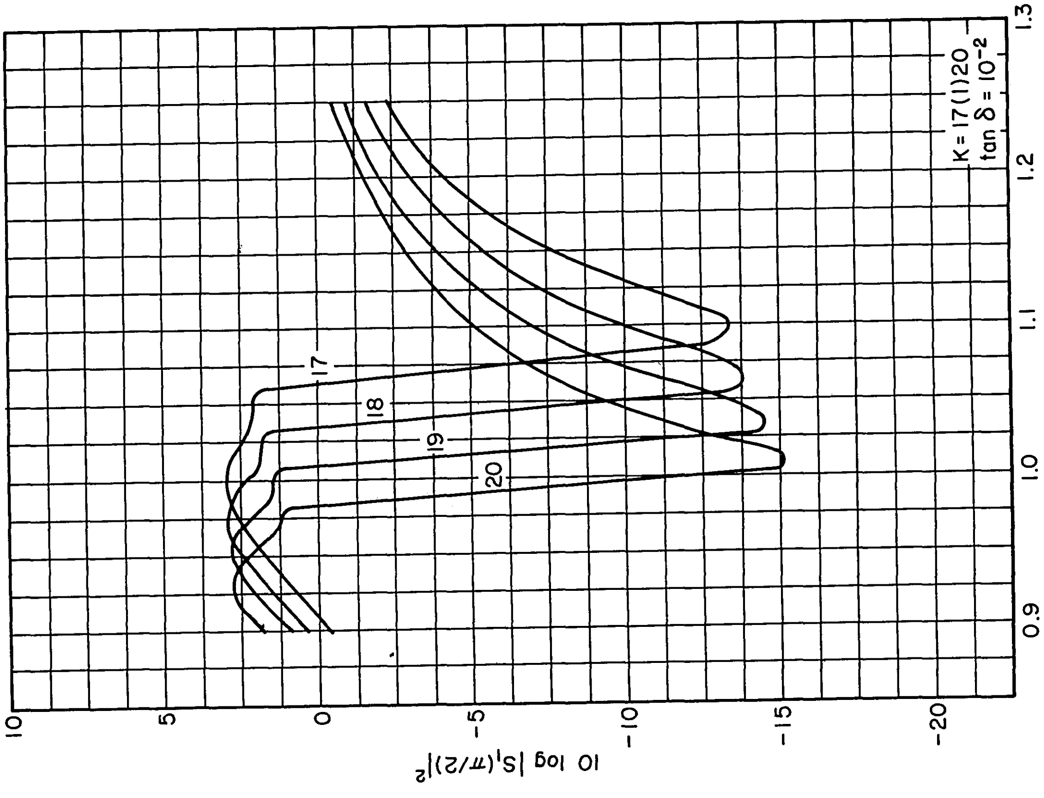


Figure 16

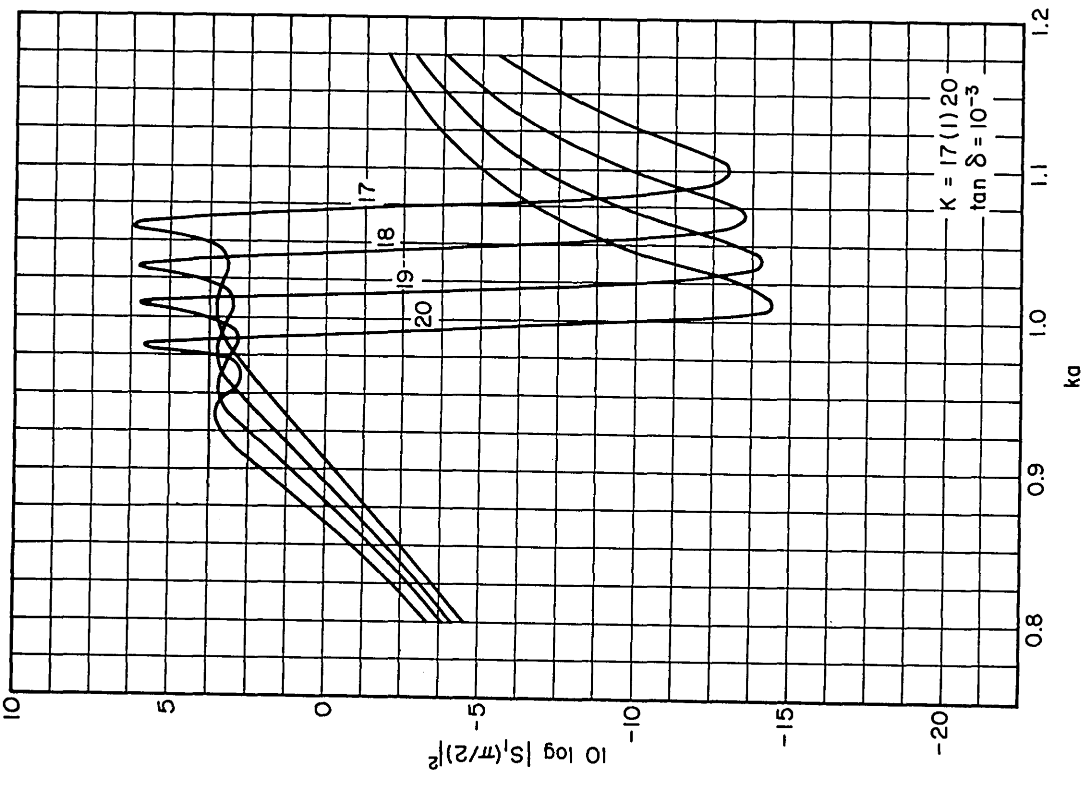


Figure 17

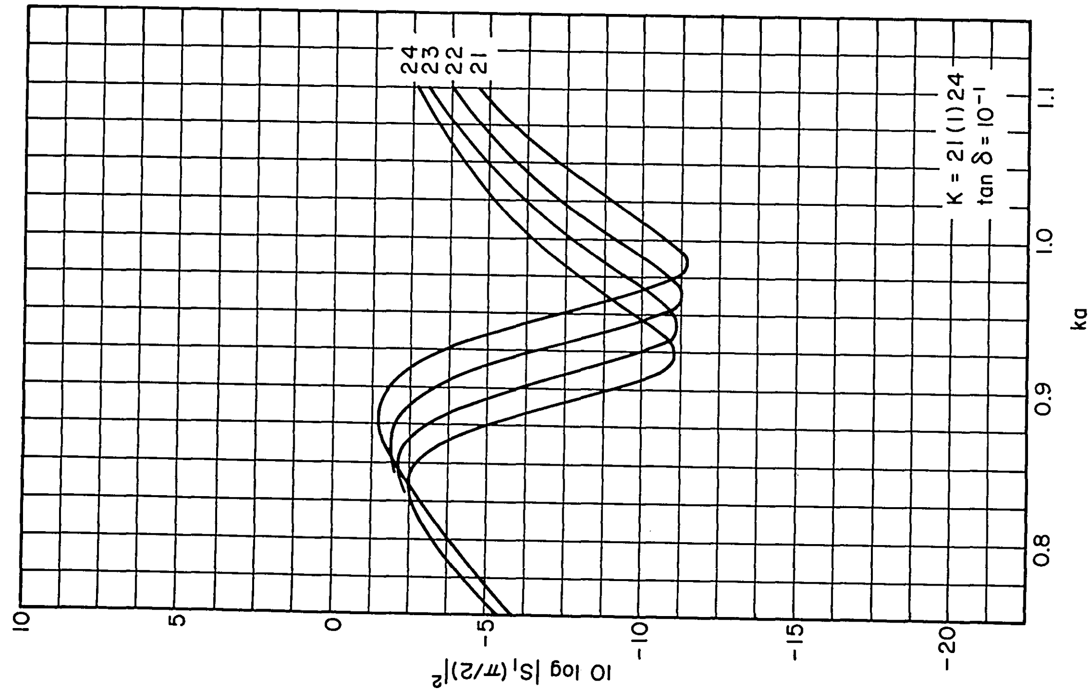


Figure 18

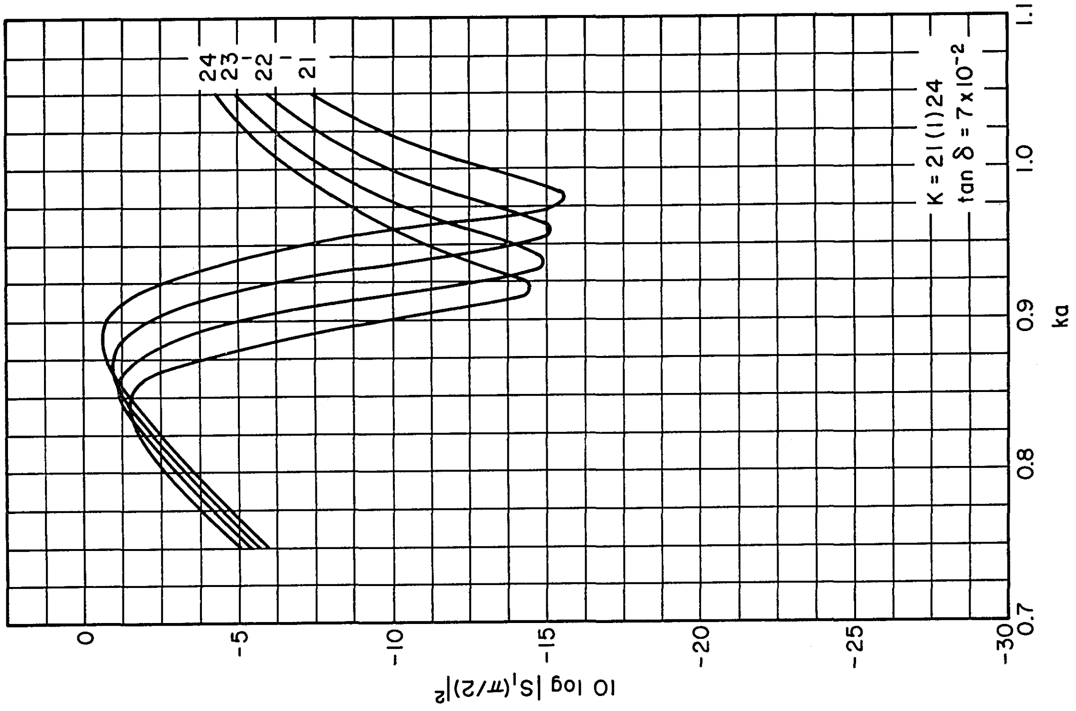


Figure 19

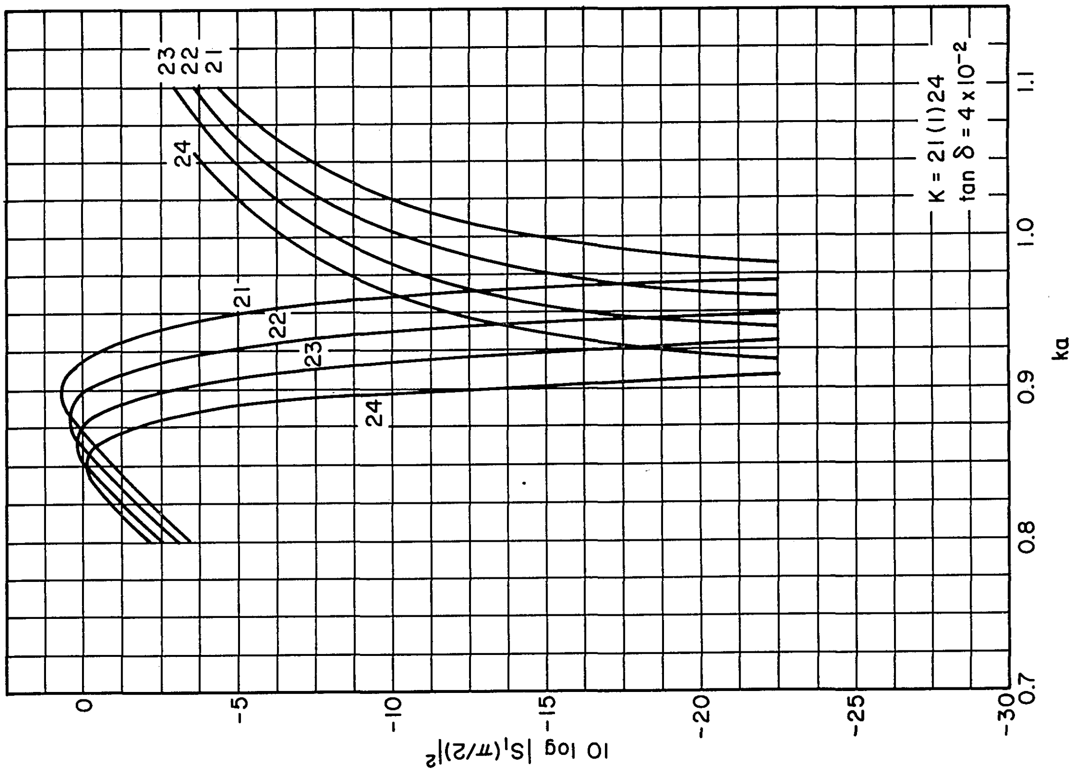


Figure 20

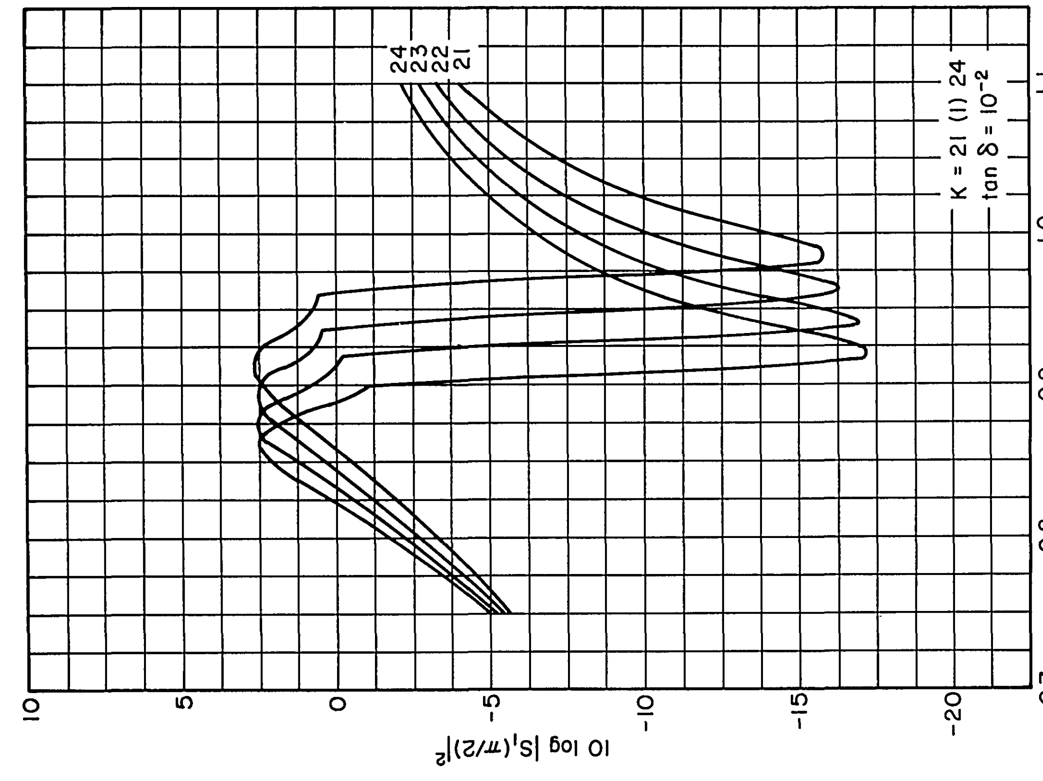


Figure 21

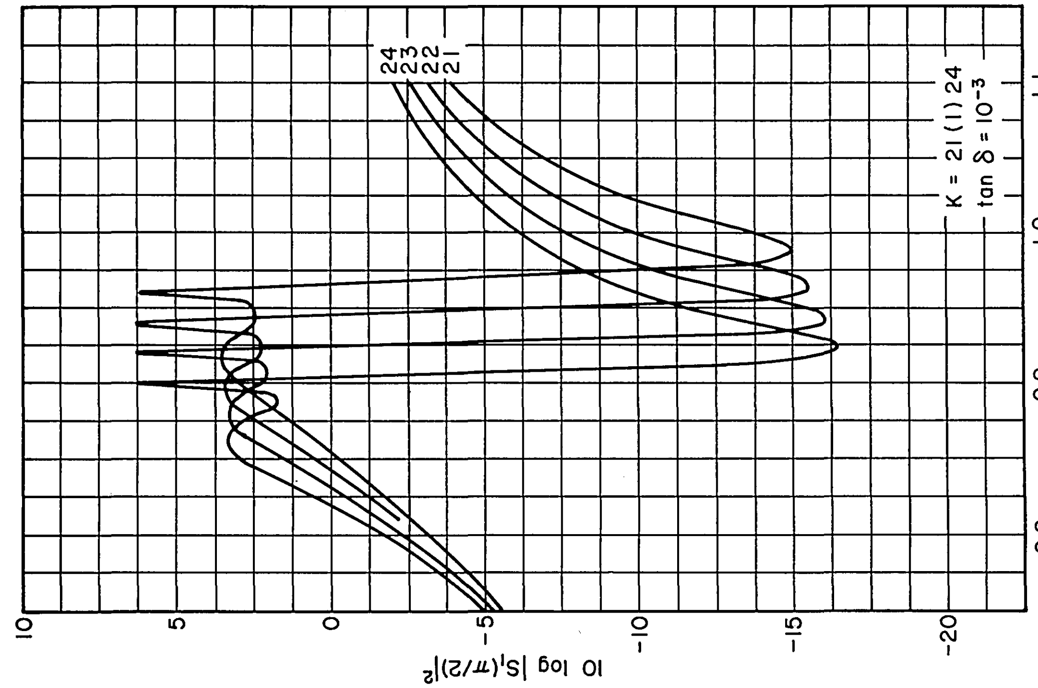


Figure 22

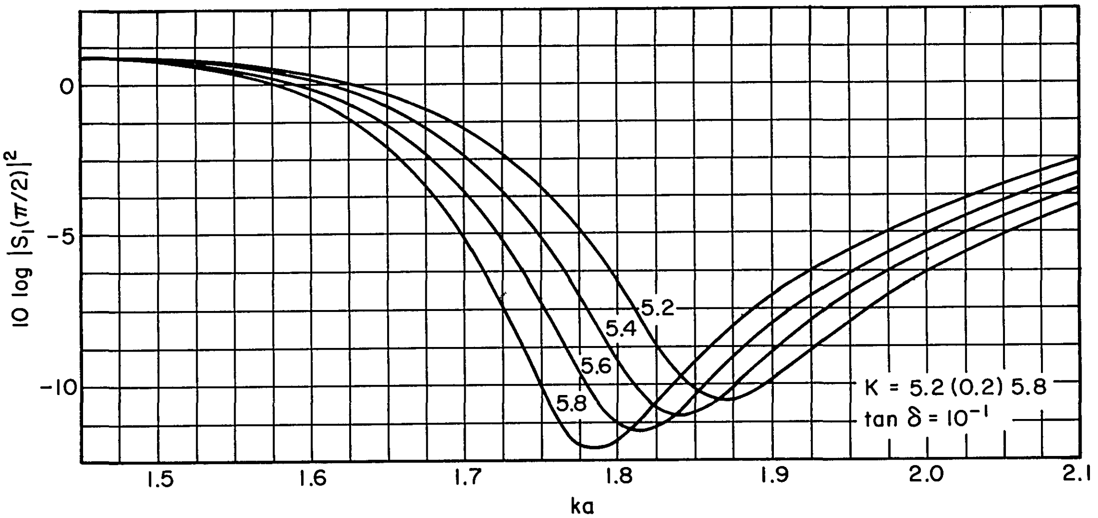


Figure 23

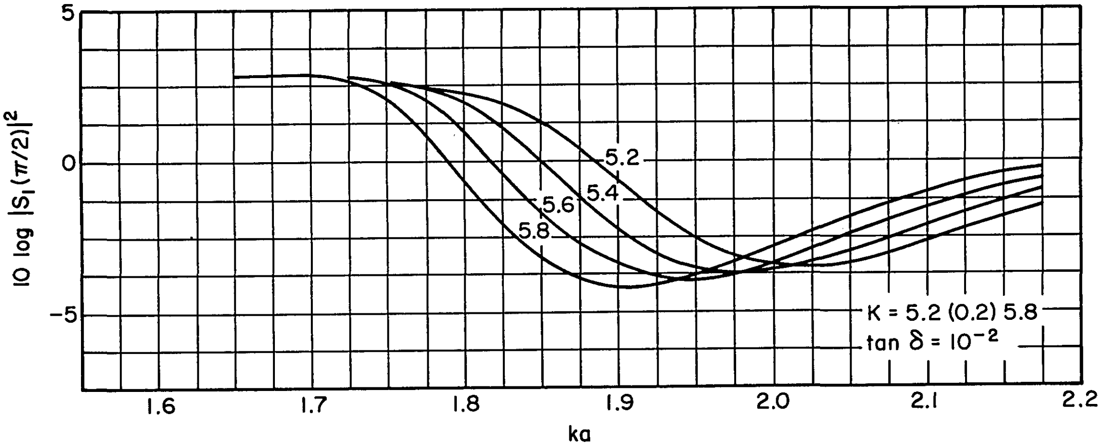


Figure 24

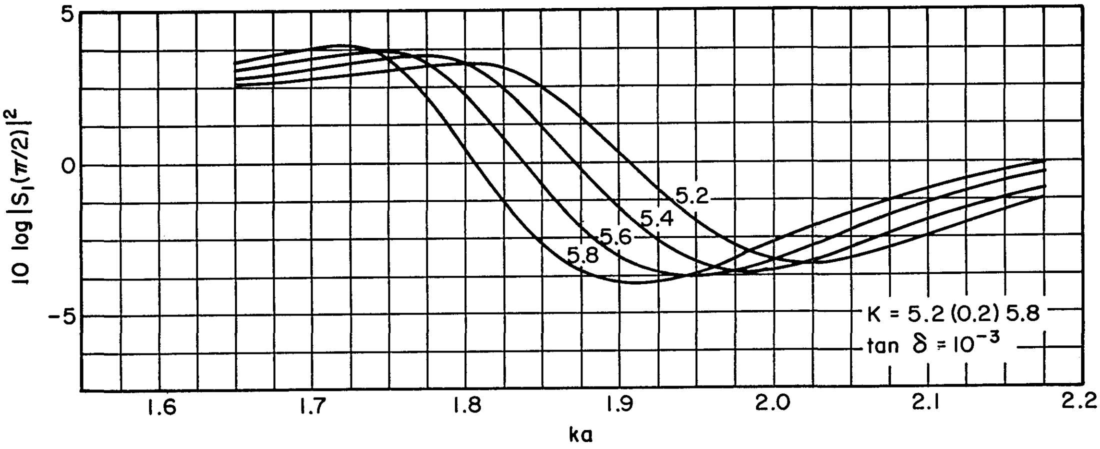


Figure 25

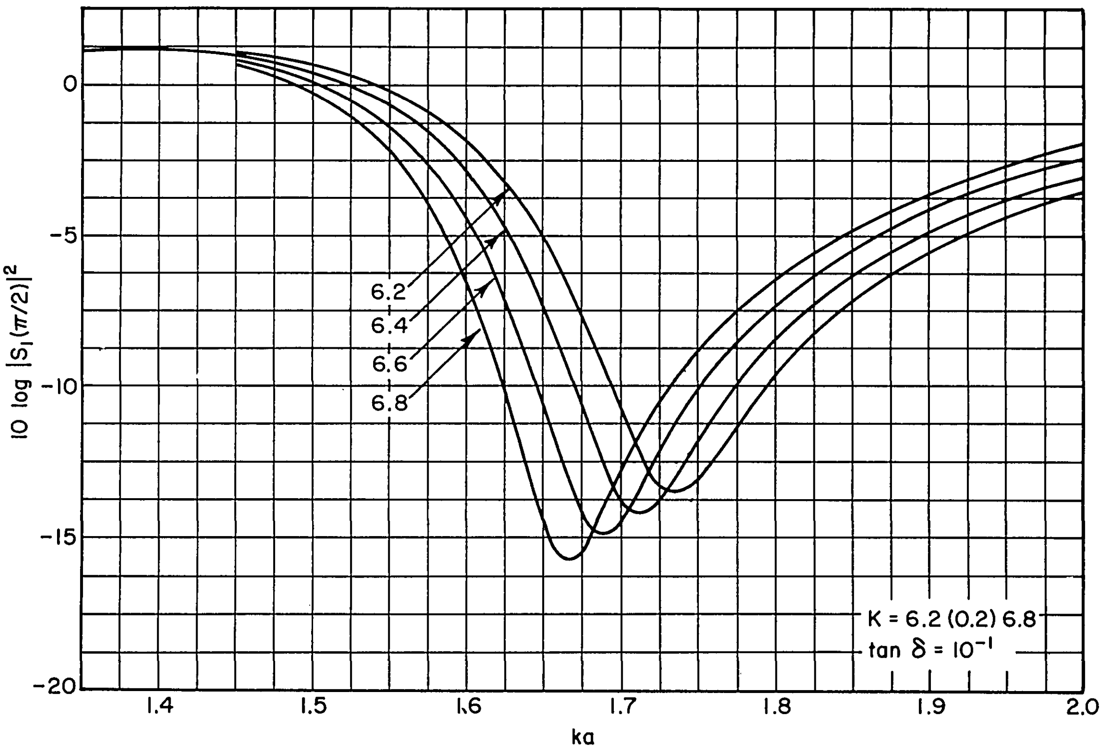


Figure 26

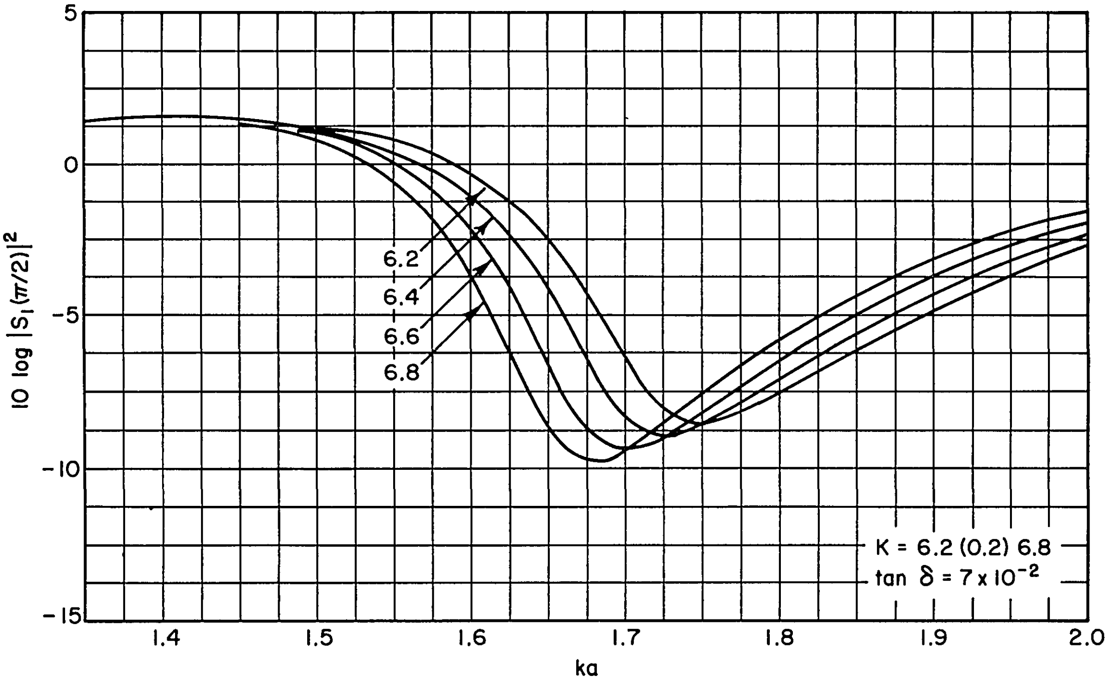


Figure 27

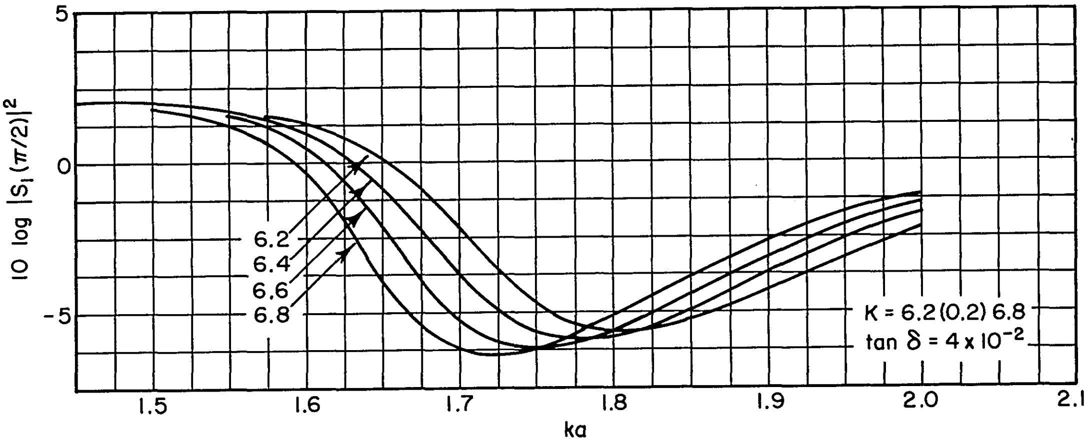


Figure 28

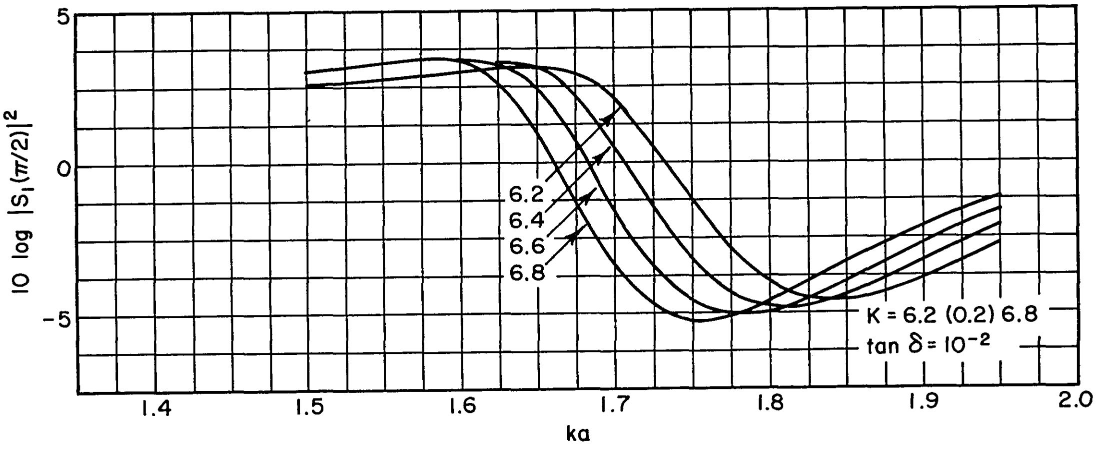


Figure 29

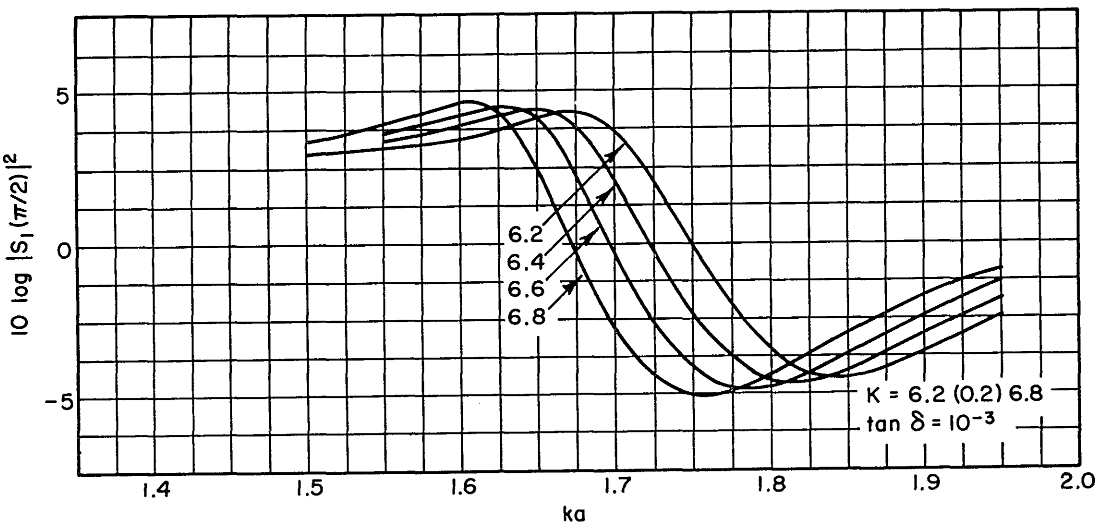


Figure 30

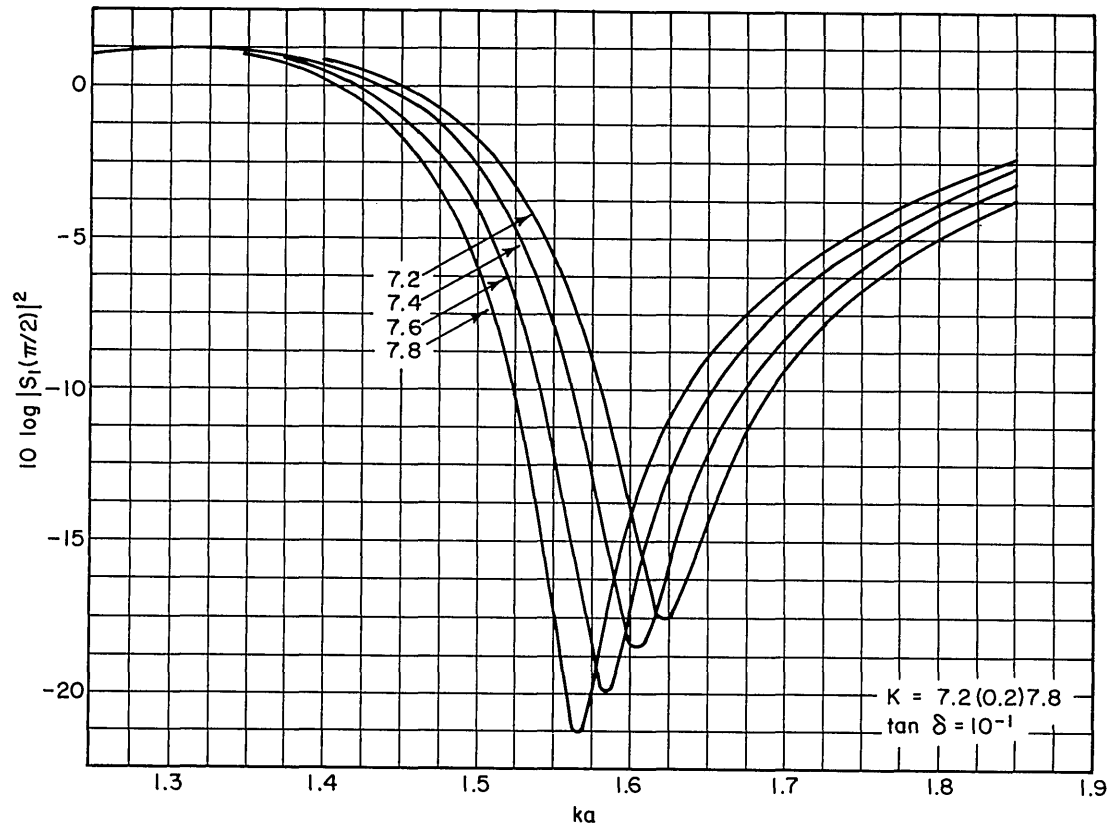


Figure 31

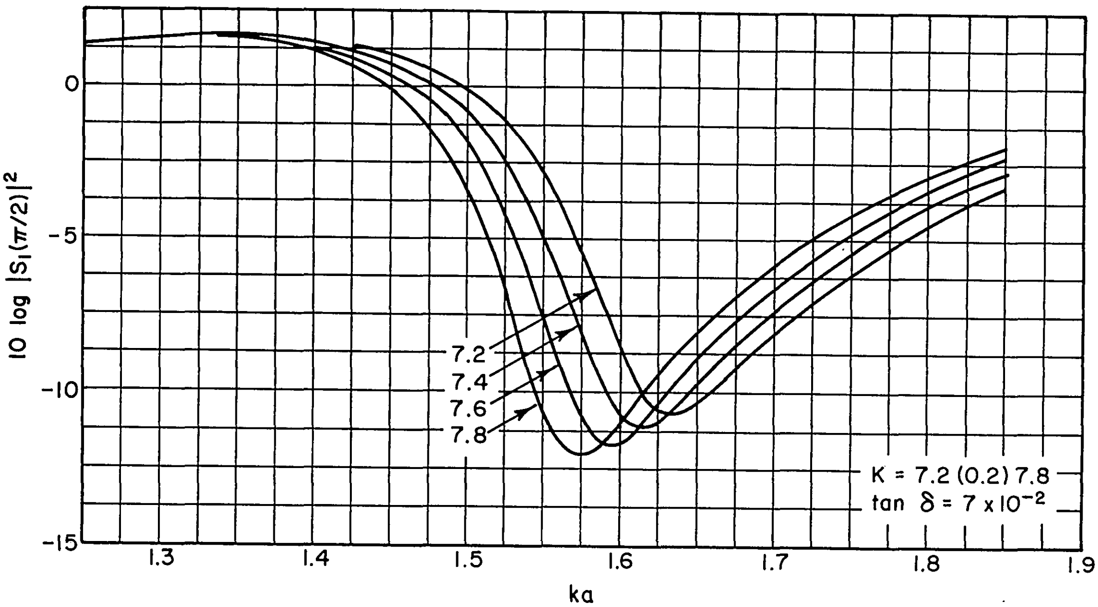


Figure 32

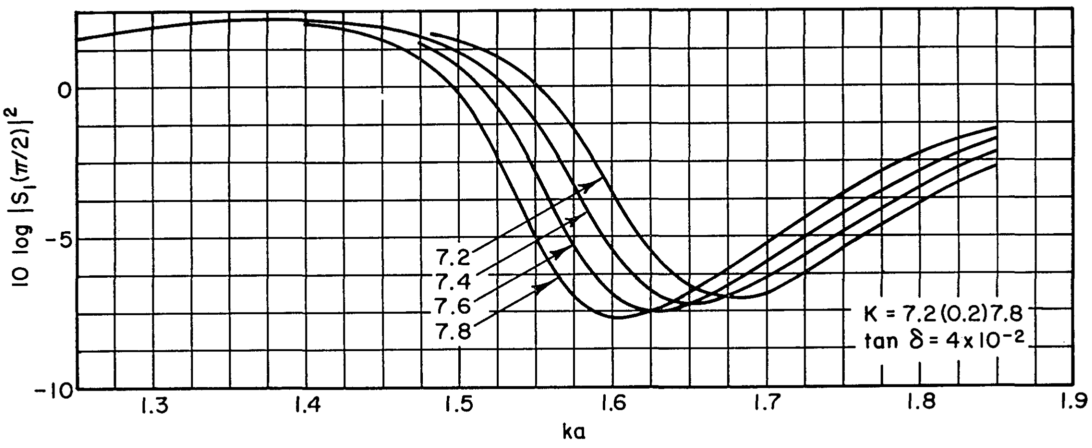


Figure 33

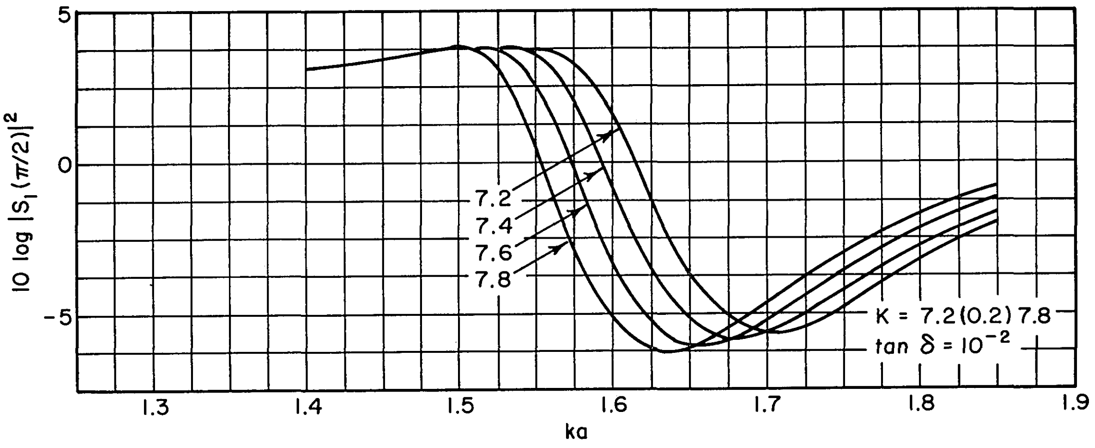


Figure 34

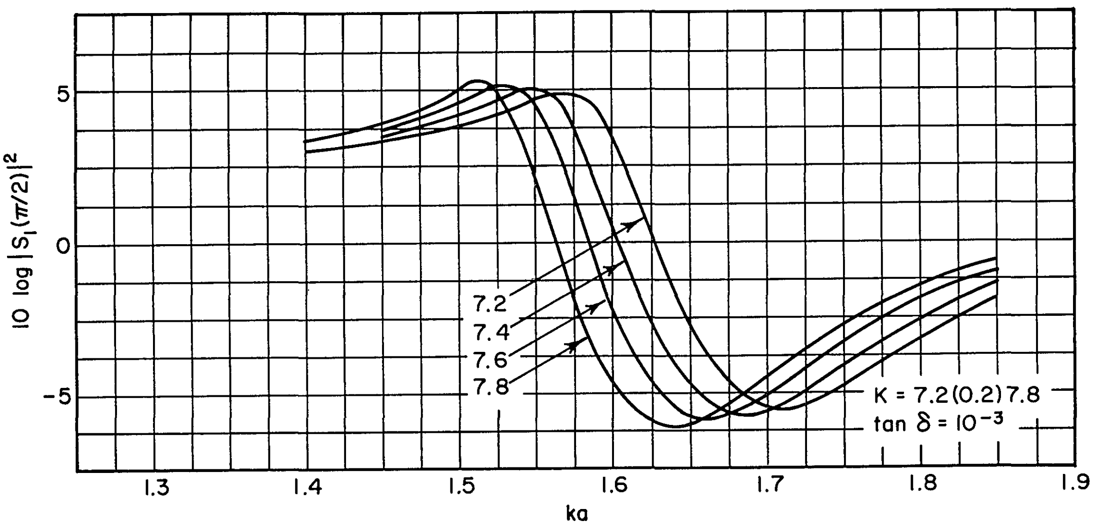


Figure 35

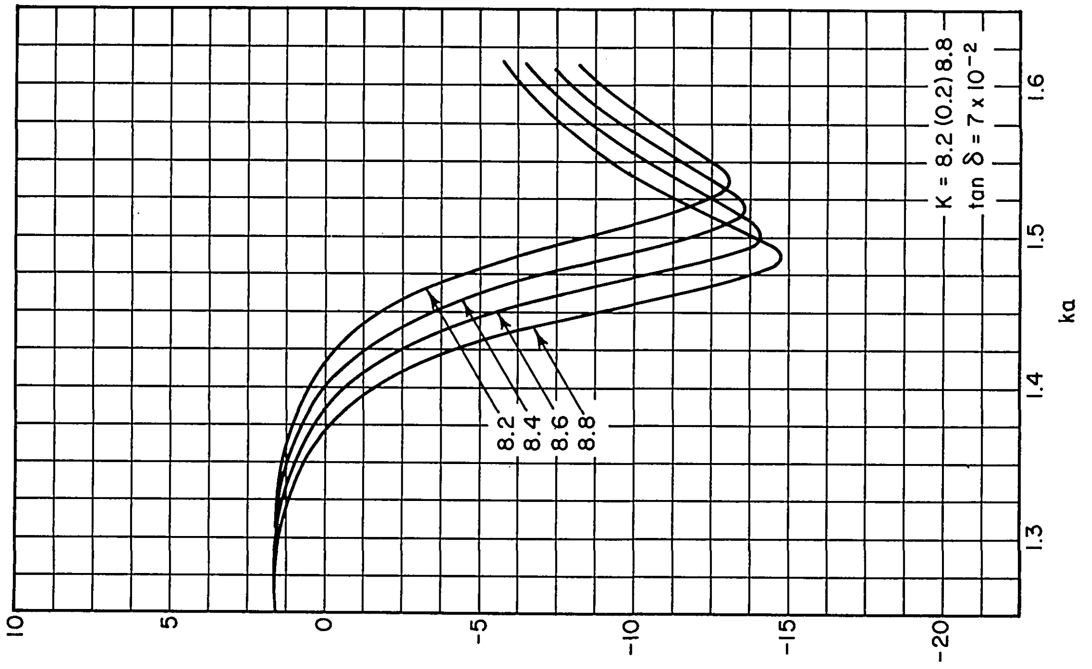


Figure 37

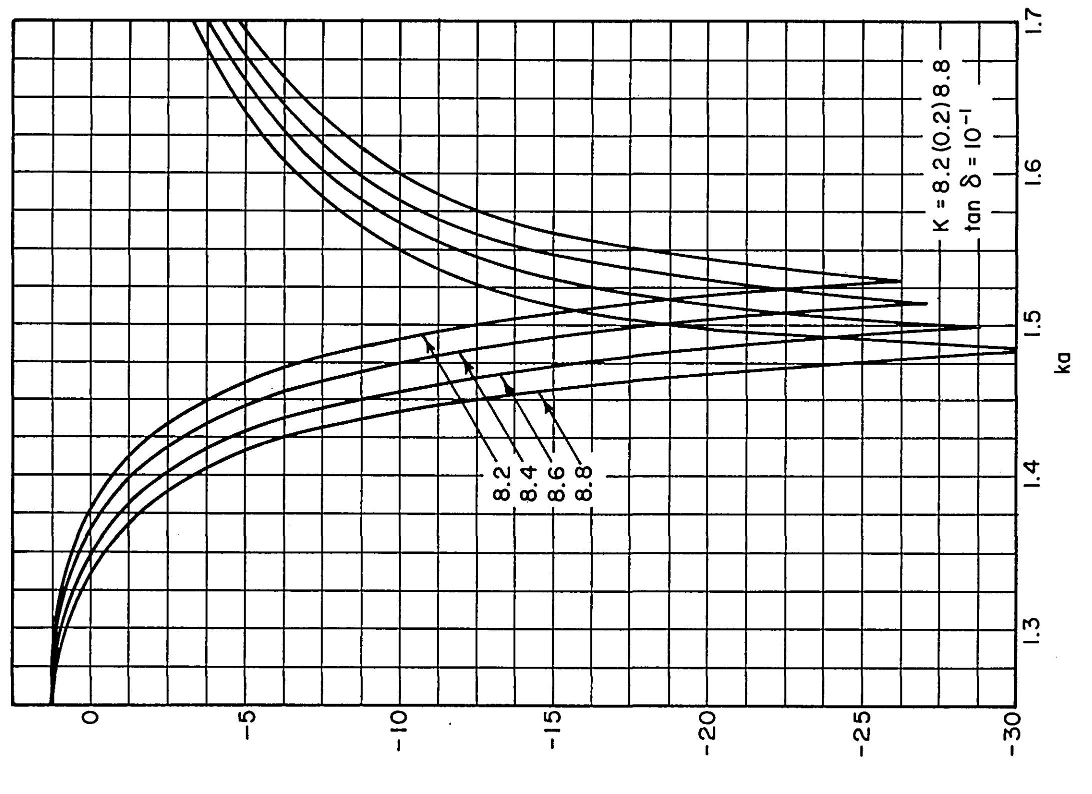


Figure 36

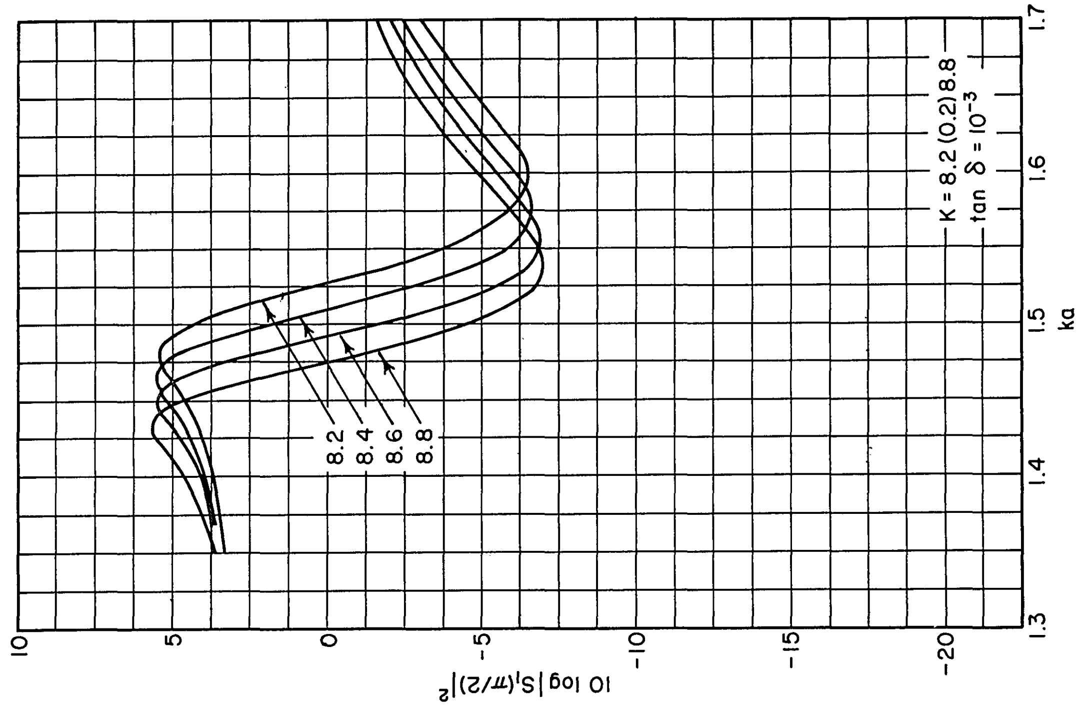


Figure 40

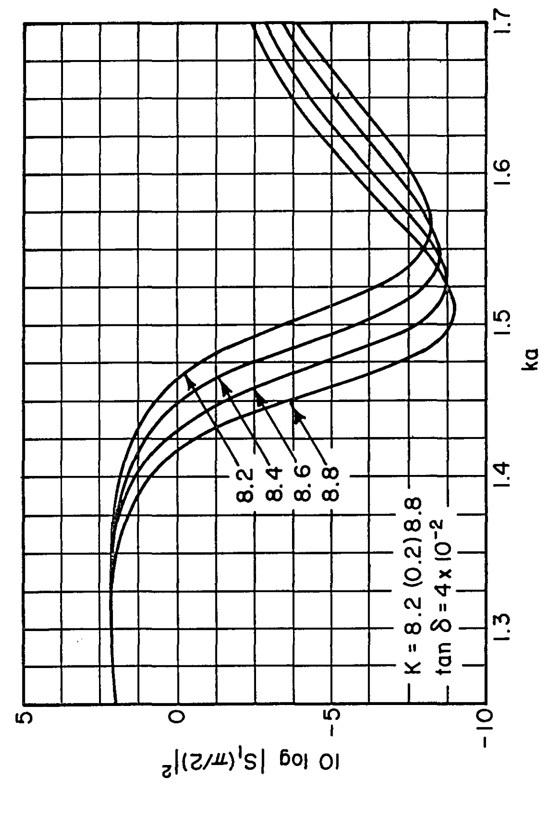


Figure 38

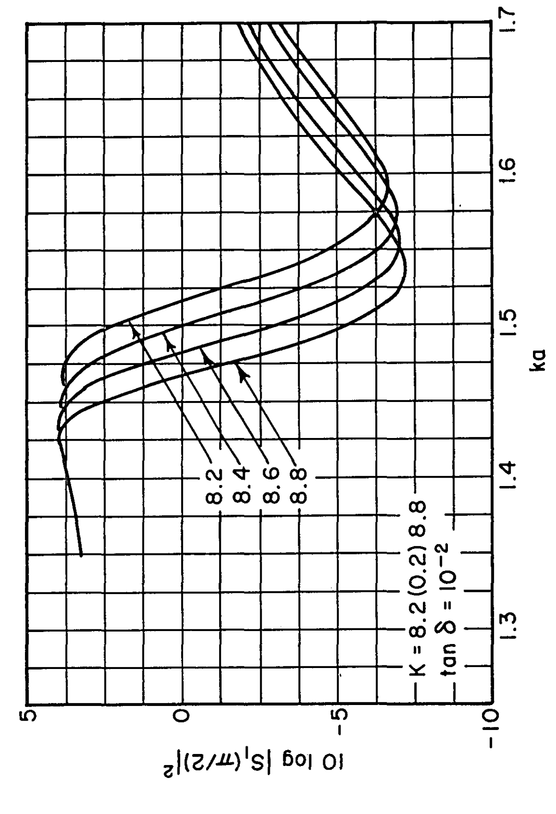


Figure 39

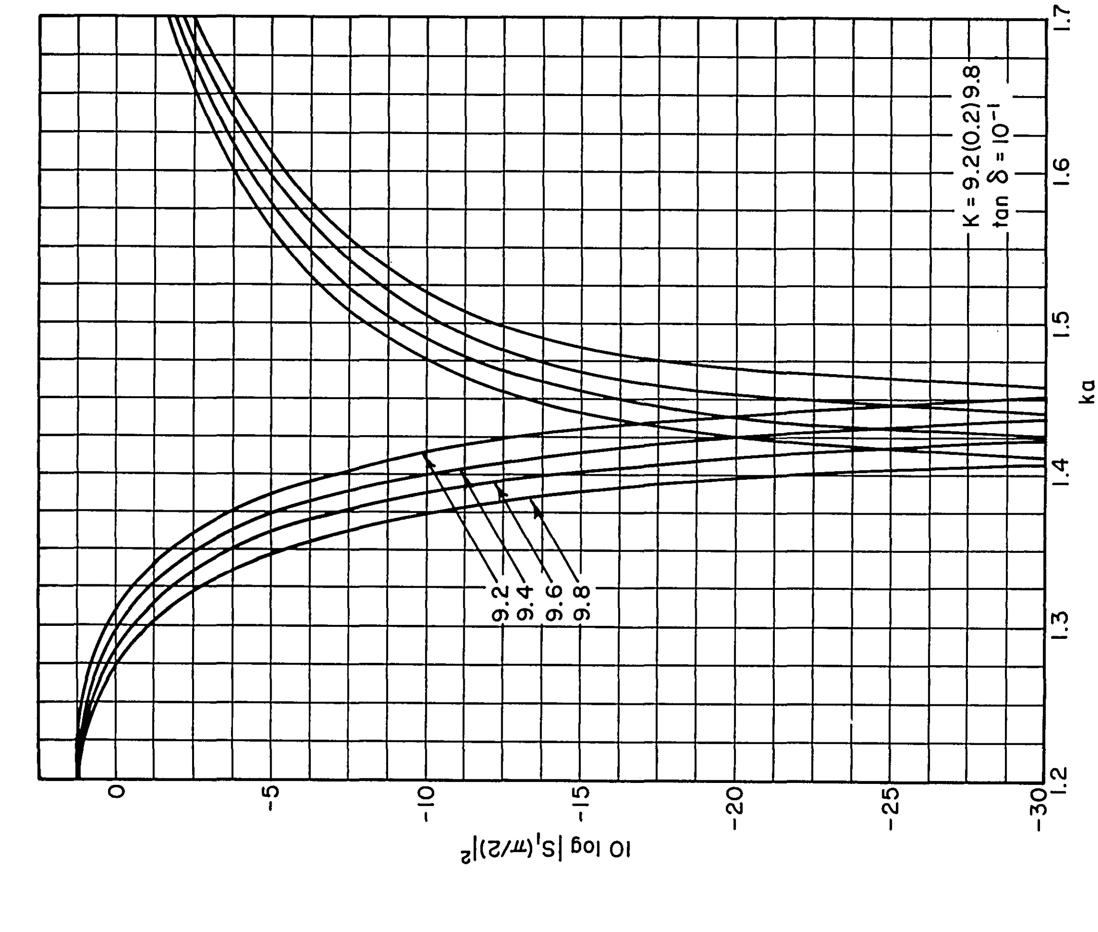


Figure 41

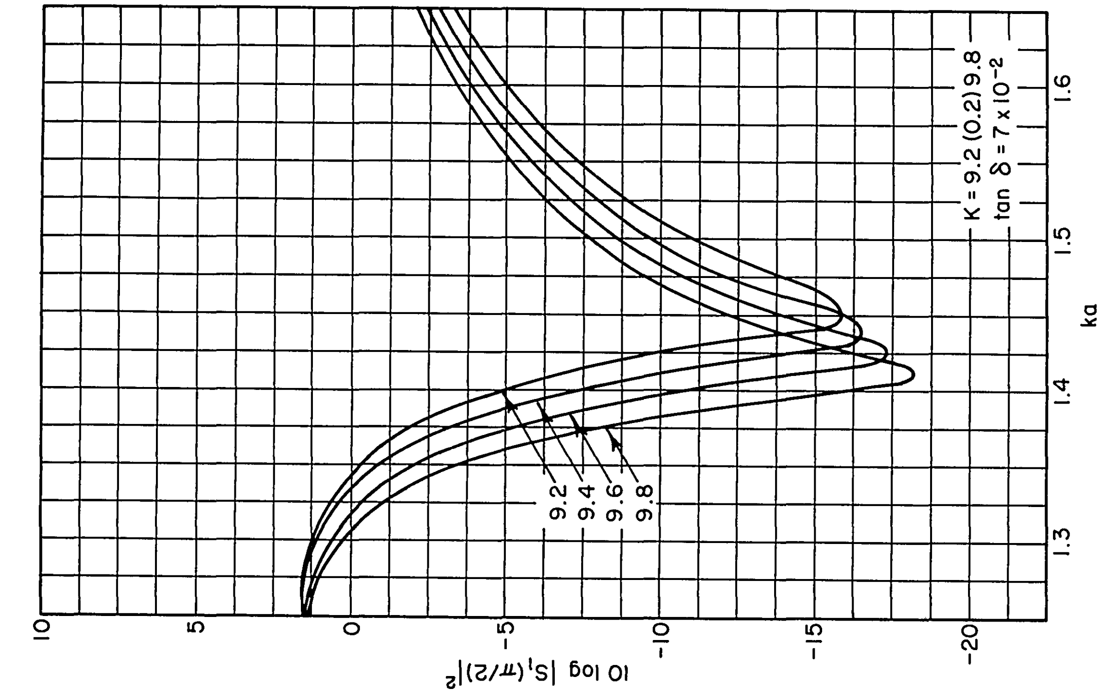


Figure 42

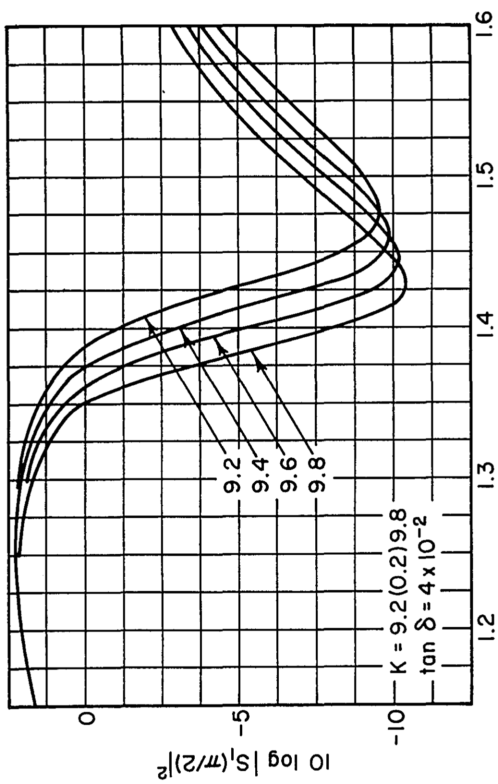


Figure 43

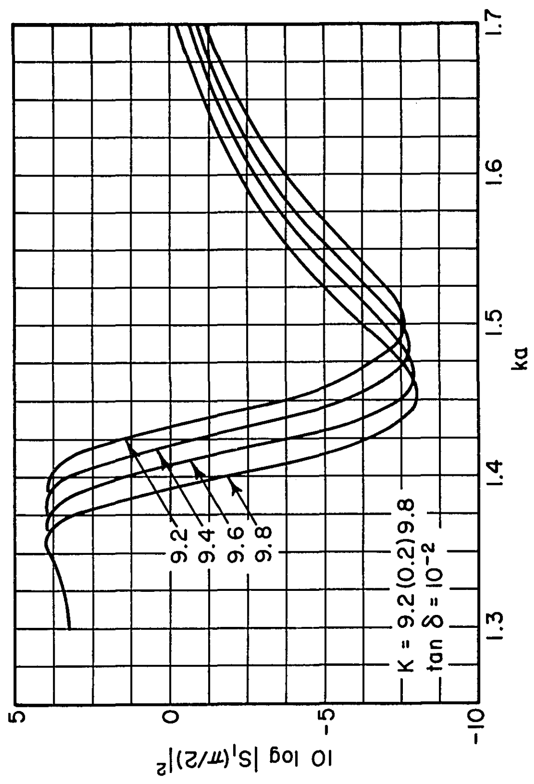


Figure 44

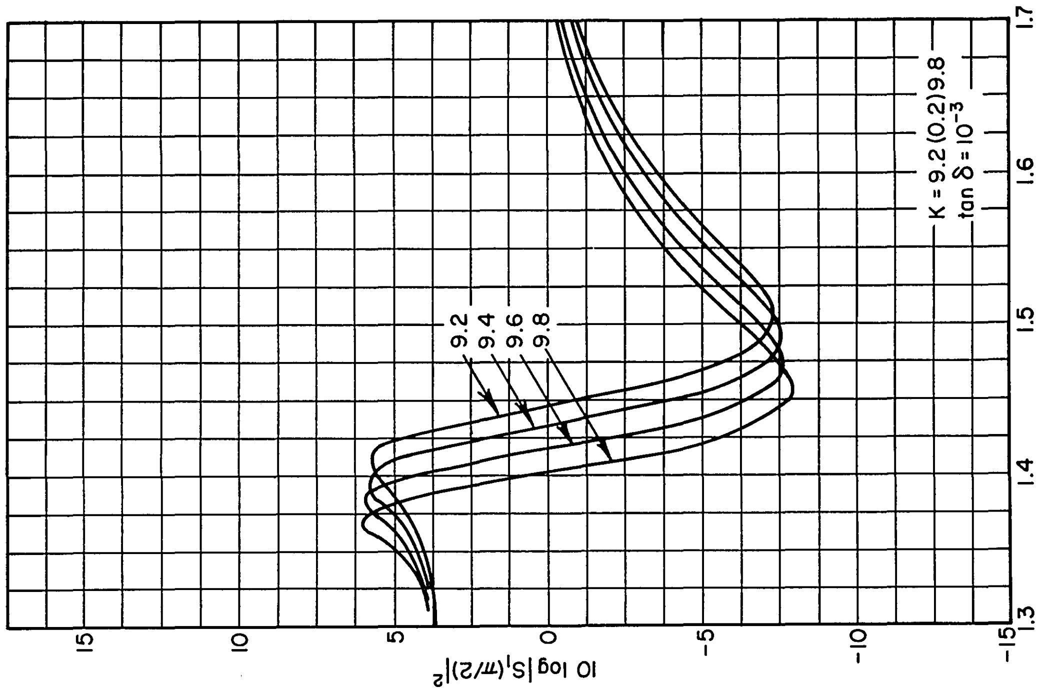


Figure 45

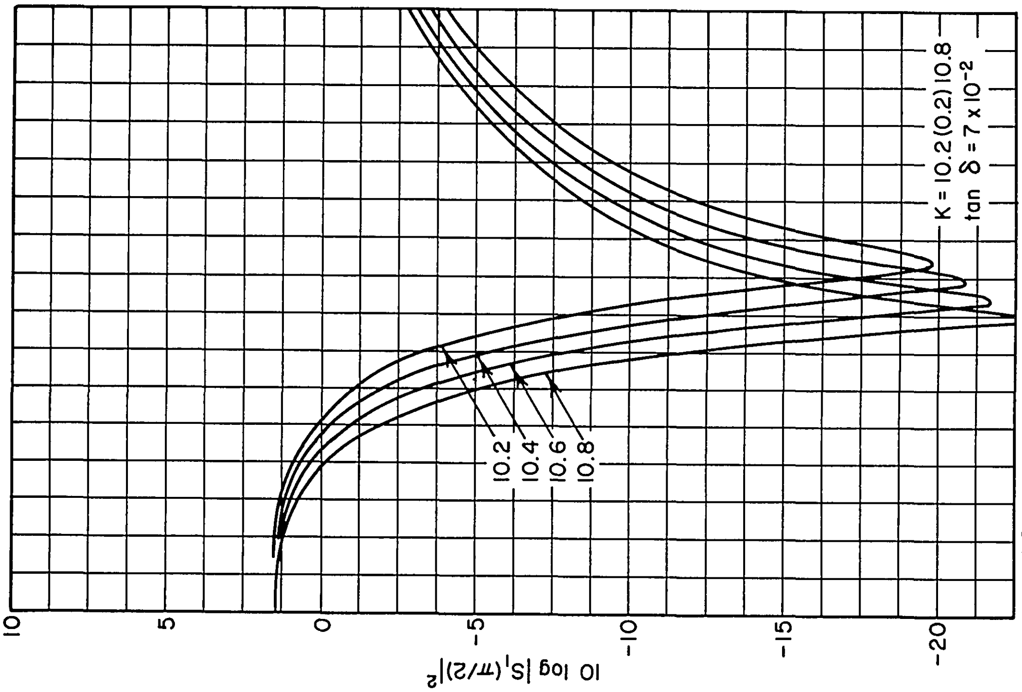


Figure 47

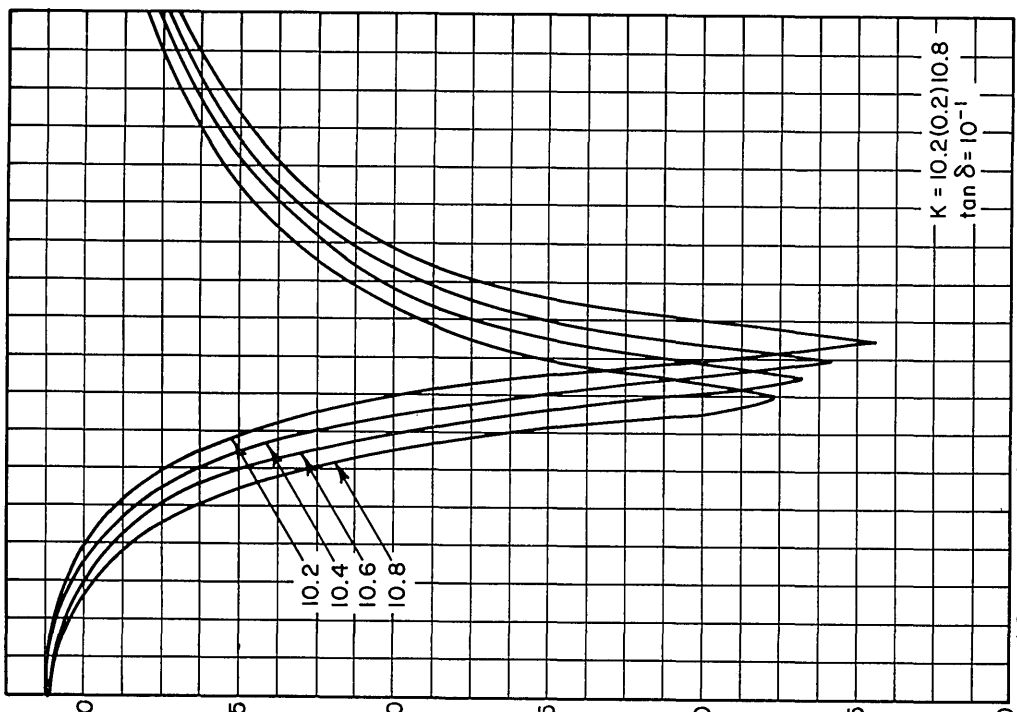


Figure 46

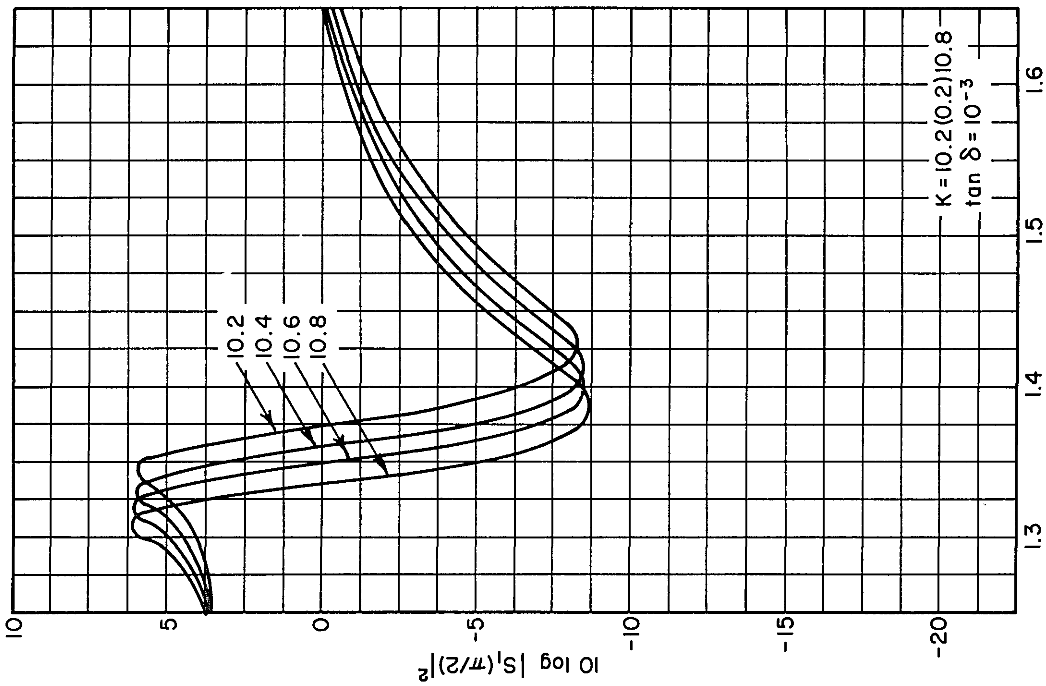


Figure 50

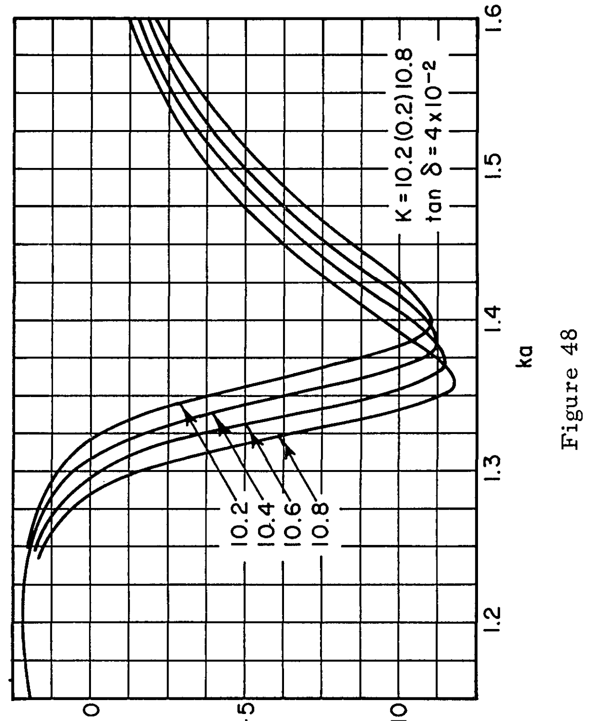


Figure 48

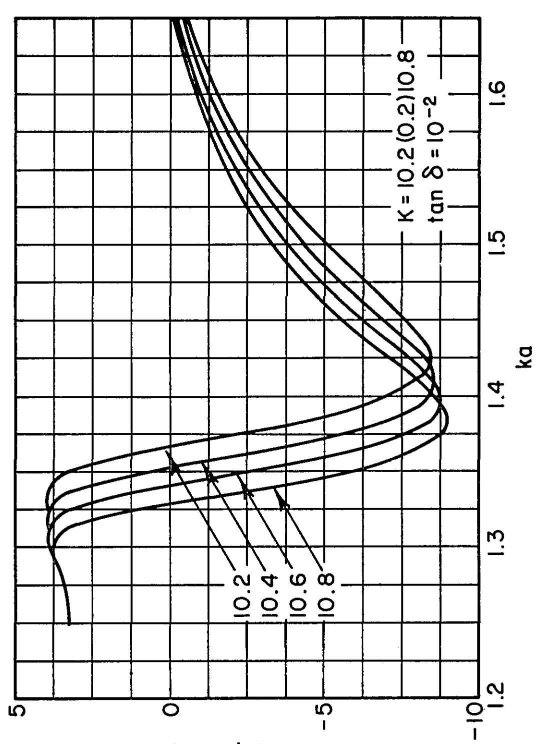


Figure 49

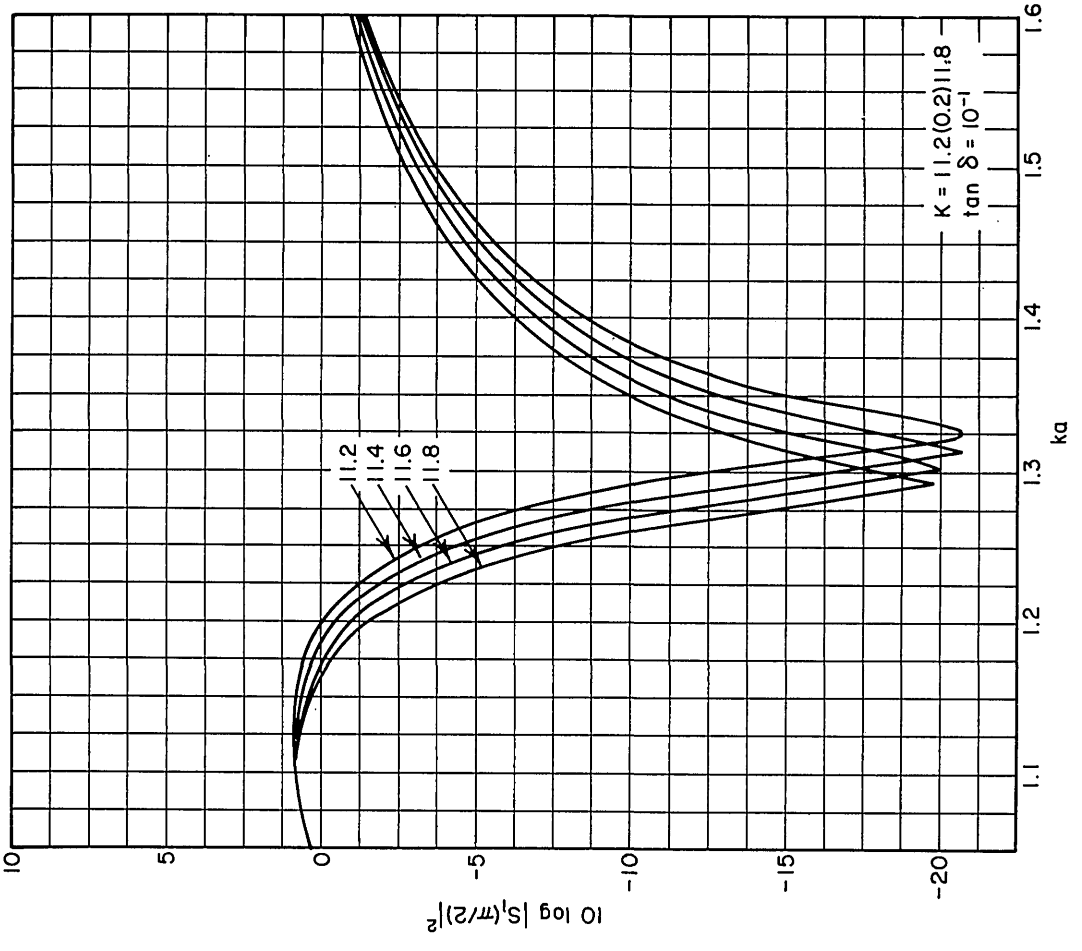


Figure 51

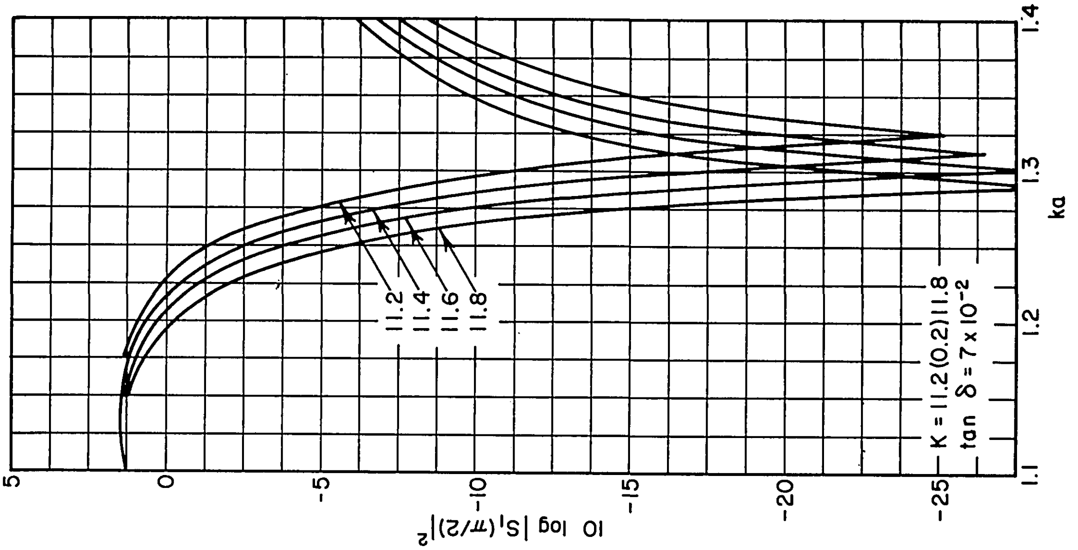


Figure 52

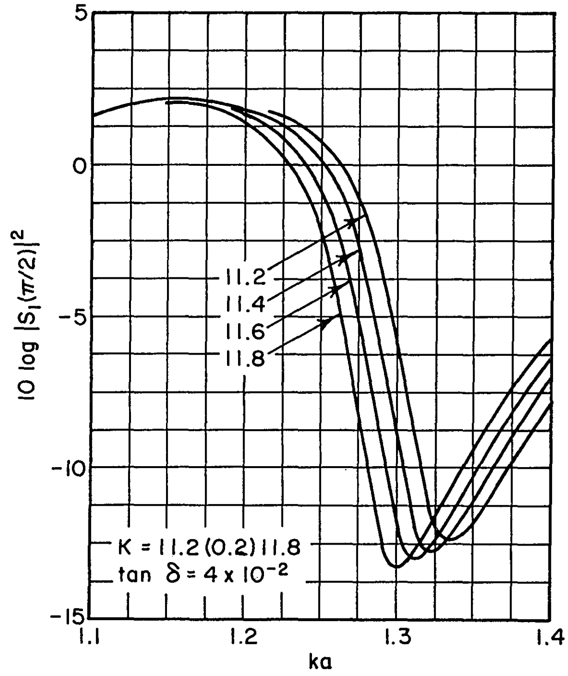


Figure 53

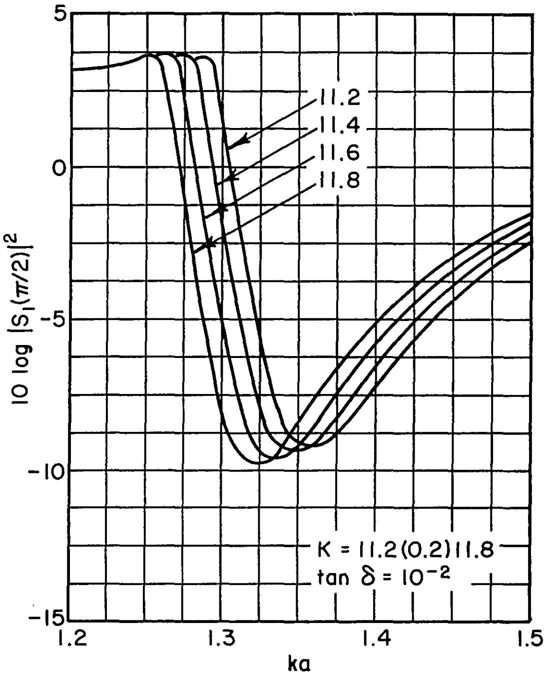


Figure 54

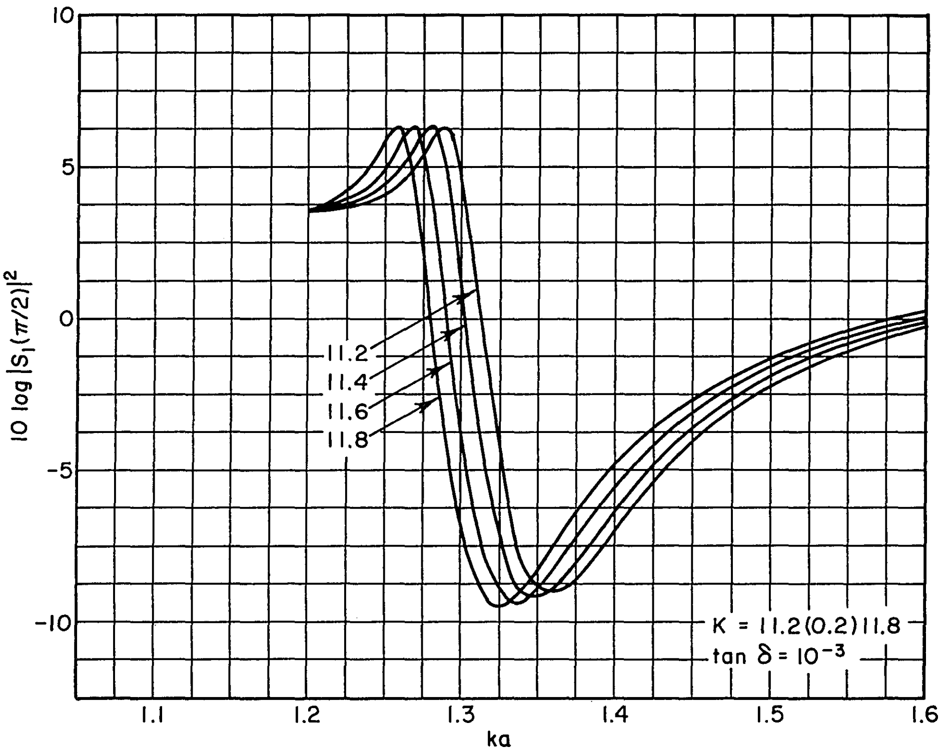


Figure 55

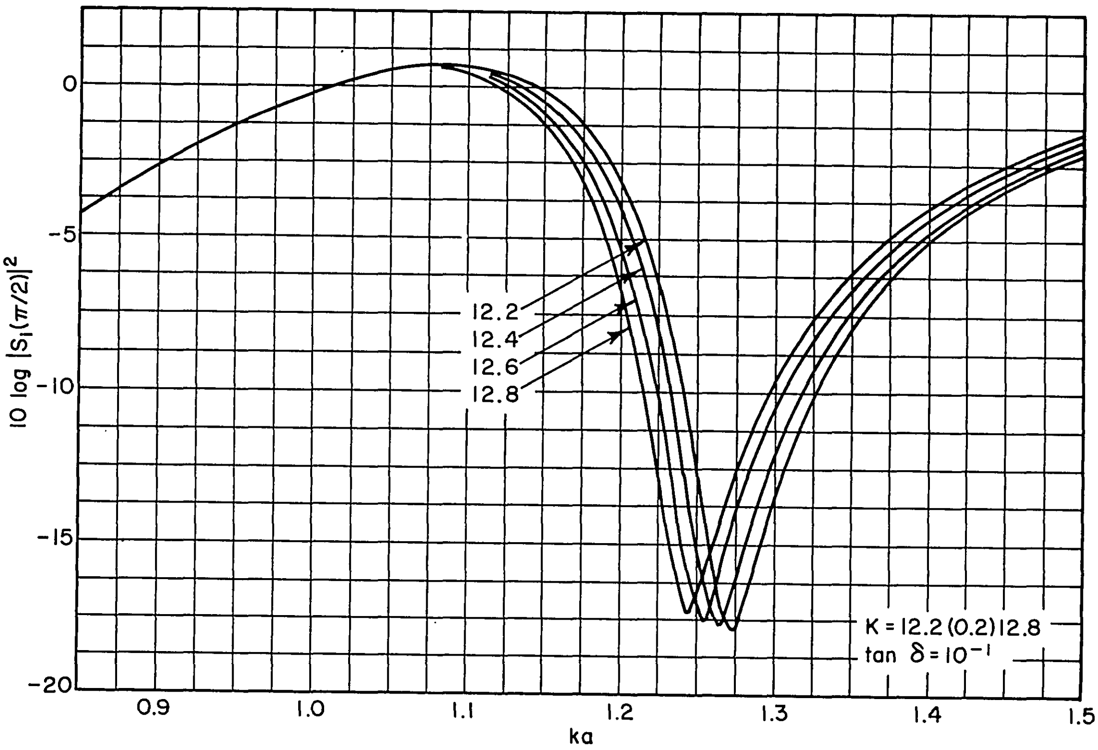


Figure 56

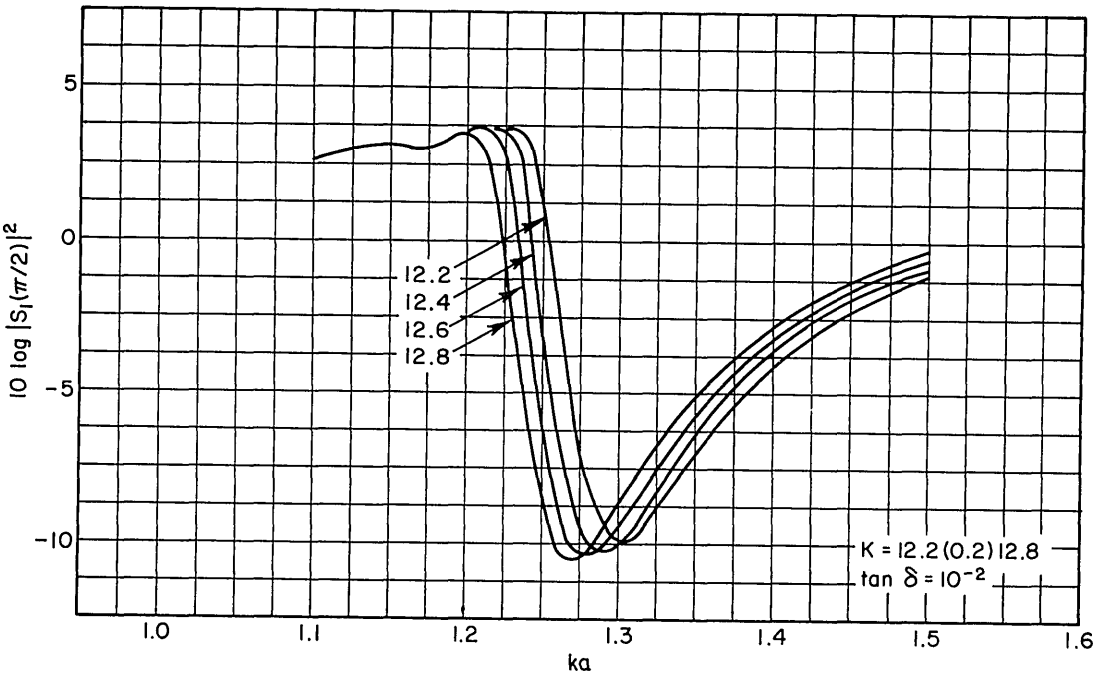


Figure 57

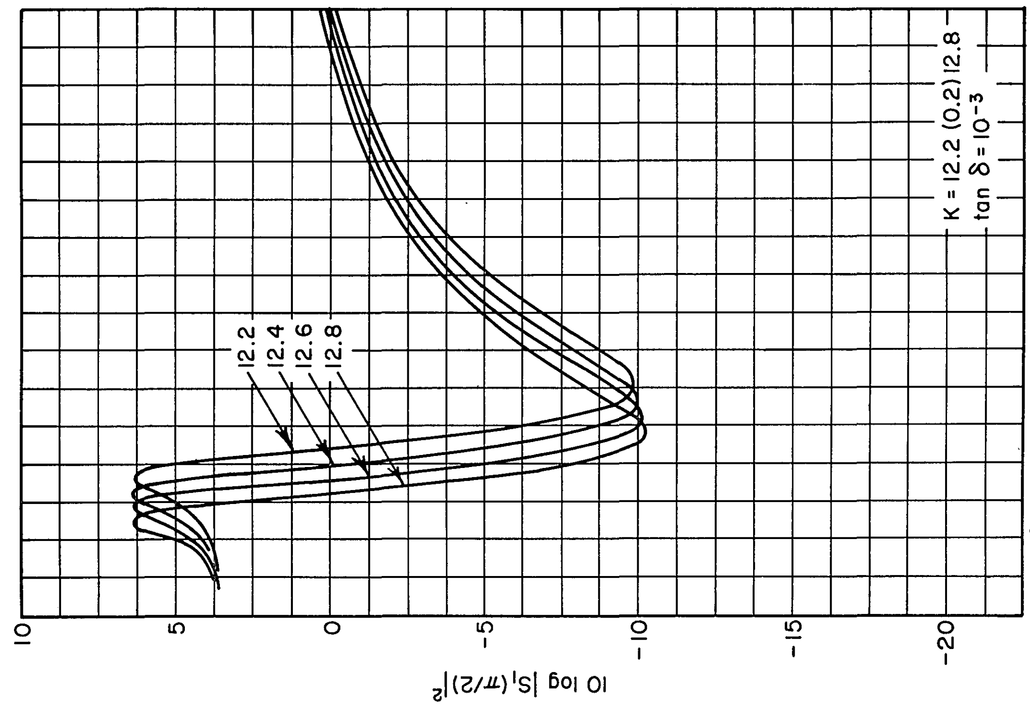


Figure 58

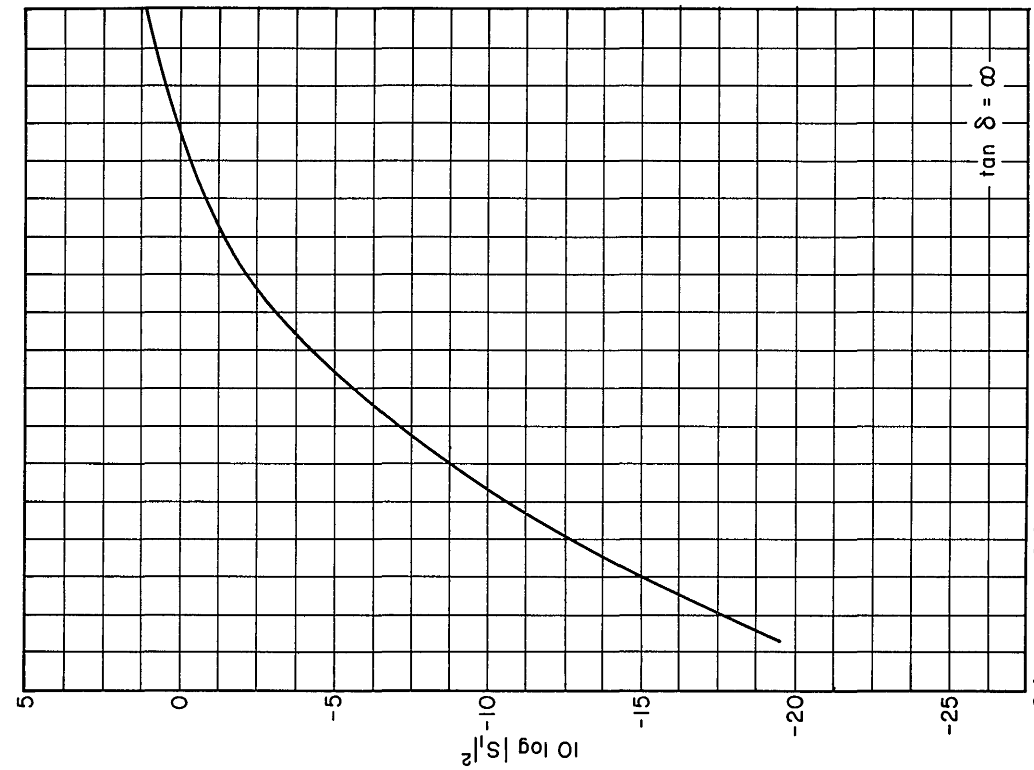


Figure 59

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* U.S. Naval Research Laboratory Washington, D.C. 20390	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	
	2b. GROUP	
3. REPORT TITLE VALUES FOR SCATTERING BY DIELECTRIC SPHERES IN THE REGION OF THE FIRST RESONANCE		
4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)* A final report on one phase of the problem.		
5. AUTHOR(S) *(Last name, first name, initial)* Keller, W.C.		
6. REPORT DATE December 30, 1965	7a. TOTAL NO. OF PAGES 39	7b. NO. OF REFS 4
8a. CONTRACT OR GRANT NO. NRL Problem P02-04 b. PROJECT NO. SF 010-02-01-9296 c. SF 010-02-01-9300 d.	9a. ORIGINATOR'S REPORT NUMBER(S) NRL Report 6365 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*	
10. AVAILABILITY/LIMITATION NOTICES Distribution of this report is unlimited. Available from CFSTI - $2.00.		
11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Dept. of the Navy (Bureau of Ships)	

13. ABSTRACT

The scattering intensity as a function of sphere size in wavelengths was plotted for H-plane scattering at 90° in the region of the first resonance. Graphs of K (the dielectric constant) from 5 to 24 and tan δ (the loss tangent) from 0.001 to 0.2 are presented for nonmagnetic spheres in vacuo. The purpose of these data is to serve as a comparison standard in evaluating K and tan δ by scattering measurements on a spherical sample. The experimental method is described in "A Method of Determining the Dielectric Constant Applicable to High Temperatures," published in the "Proceedings of the OSU-RTD Electromagnetic Window Symposium," Vol. III, June 1964.

DD FORM 1 JAN 64 1473

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
High-temperature dielectric constant						
Dielectric properties						
Determination						
Measurement						
Spheres						
Electromagnetic waves						
Polarization						
Scattering						
Intensity						
High-temperature research						

INSTRUCTIONS

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization *(corporate author)* issuing the report.

2*a*. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2*b*. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7*a*. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7*b*. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8*a*. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8*b*, 8*c*, & 8*d*. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9*a*. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9*b*. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers *(either by the originator or by the sponsor)*, also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through
_______________________________."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through
_______________________________."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through
_______________________________."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring *(paying for)* the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS)*, *(S)*, *(C)*, *or (U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.